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Micro-milling stability indicator by pseudo-inverse of time series

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Abstract

This study presents an indicator to estimate the absolute stability of the micro-milling process using the time series data. In general, the milling process is widely modelled by delay differential equations (DDE). For better modelling, accurate information about cutting coefficients and dynamic response of the tool and workpiece is required. The resultant Floquet transition matrix of governing DDE is then estimated and the eigenvalues of this transition matrix are a robust indicator of chatter. However, in this study the presented indicator is based on estimating the Floquet transition matrix from the simulated time series data of the cutting process. Appropriate sampling of the simulated data and correct use of pseudo-inverse of the time series data is employed to calculate the Floquet transition matrix. The presented method can also be used to identify the chatter from the data recorded during machining. When implemented in real-time, the indicator can be used as a robust tool for chatter detection and subsequent avoidance. The presented indicator can be proven superior to stability indicators based on variance of the process data, especially when chatter frequency coincides with the integer multiple of the tooth passing frequency.

Keywords: milling stability, regenerative chatter, delay differential equation (DDE), Floquet transition matrix

1. Introduction

The milling process is widely modelled using delay differential equations (DDEs). Stability investigation of DDEs is often carried out by numerical simulations [1]. Robust stability investigation methods involve the construction of the Floquet transition matrix from the governing differential equations of the process. The semi-discretisation method to calculate the appropriate coefficient matrix and transition matrix was composed and applied for the dynamics of the cutting process by Insperger and Stéphán [2] and with discrete delay it is presented by the same authors in [3]. Other analytical methods to predict the stability for given cutting conditions were proposed and made popular by Altintas and Budak [4].

Chatter prediction methods may suffer from modelling inaccuracies, and the predicted stability limit may differ from real limit. With the improvements in machine hardware and available computation power, real-time chatter avoidance has become possible [5]. Chatter avoidance, in terms of successful detection and control, can offer added advantages over standard chatter prediction techniques. For chatter avoidance to be effective, a robust chatter indicator is required. Real-time chatter indicators should be easy to implement and preferably computationally 'cheap' and fast. One such category of indicators rely on the statical calculations on the time series data recorded during machining. These indicators exploit a sharp change in variance or standard deviation of the recorded signal as the process turns unstable [6].

The presented work focuses on the estimation of the transition matrix using the time series data for micro-milling. The eigenvalues of the identified transition matrix are the indicator of process stability. By the very definition of the stable system, if any of the eigenvalues of the estimated transition matrix lies outside the unit circle, the process is deemed unstable. Unlike the relative nature of statistical indicators, where a numerical limit on the indicators is fixed by the user (e.g. 1 µm on standard deviation of tool displacement in [6]), the presented indicator is absolute.

The next section outlines the theoretical background of the involved DDE and estimation of transition matrix from the time series data, the section afterwards presents an example.

2. Transition matrix

Consider a linear DDE

$$\ddot{x}(t) + 2\zeta\dot{x}(t) + (1+p)x(t) = px(t-\tau) \tag{1}$$

The above equation models the milling process [7]. Stability of the above differential equation is studied in the (τ, p)-space and can be expressed in closed-form. All the points below the bounding curve are stable operating points, this plot is commonly known as the stability lobe diagram (SLD), where the delay τ is analogous to the spindle speed S and variable p to the depth of cut a_p.

Taking the Laplace transform of (1) results in

$$(s^2 + 2\zeta s + 1 + p)\mathrm{L}(x) - (s+2\zeta)x_0 - v_0 = p\mathrm{L}(\phi) \tag{2}$$

where, ϕ is initial function and x_0 and v_0 are the initial displacement and velocity respectively (at $t = 0$). For the first interval $(0,\tau]$ the laplace $\mathcal{L}(x)$ can be represented as:

$$\mathrm{L}(x) = \frac{p\mathrm{L}(\phi) + g(s, x_0, v_0)}{h(s,p)}$$

where, (3)

$$g(s, x_0, v_0) = (s+2\zeta)x_0 + v_0$$
$$h(s,p) = s^2 + 2\zeta s + 1 + p$$

Taking the inverse laplace of (3) at the end of the interval τ $(t = \tau)$ results in state (x_1, v_1), a further generalisation of the terms therafter results in the map (4). A more detailed derivation is presented in [7].

In the equation (4), the state (x_n, v_n) depends on all the previous states and describe an infinite-dimensional system.

Notations of Equation (4) can be simplified with companion matrix $\boldsymbol{\mathcal{B}}_n$ in (5).

$$\begin{pmatrix} x_{n+1} \\ v_{n+1} \\ x_n \\ v_n \\ \vdots \end{pmatrix} = \begin{pmatrix} a_n & b_n & a_{n-1} & b_{n-1} & \cdots \\ c_n & d_n & c_{n-1} & d_{n-1} & \cdots \\ 1 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & \cdots \\ \cdots & \cdots & \ddots & \cdots & \cdots \end{pmatrix} \begin{pmatrix} x_n \\ v_n \\ x_{n-1} \\ v_{n-1} \\ \vdots \end{pmatrix} \quad (4)$$

$$\boldsymbol{y}_{n+1} = \boldsymbol{\mathcal{B}}_n \boldsymbol{y}_n \quad (5)$$

Increasing the system order of the discretised truncated system, the spectrum of $\boldsymbol{\mathcal{B}}_n$ tends to match that of the infinite-dimensional system. Coefficients a_i, b_i, c_i, and d_i in (4) can be read as coefficients of x_i and v_i from (6)

$$x_{n+1} = \sum_{i=1}^{n} p^i \mathrm{L}_\tau^{-1}\left(\frac{s+2\zeta}{h^{i+1}(s+p)}\right) x_{n-1} + \sum_{i=1}^{n} p^i \mathrm{L}_\tau^{-1}\left(\frac{1}{h^{i+1}(s+p)}\right) v_{n-1}$$

and (6)

$$v_{n+1} = \sum_{i=1}^{n} p^i \mathrm{L}_\tau^{-1}\left(\frac{s(s+2\zeta)}{h^{i+1}(s+p)}\right) x_{n-1} + \sum_{i=1}^{n} p^i \mathrm{L}_\tau^{-1}\left(\frac{s}{h^{i+1}(s+p)}\right) v_{n-1}$$

A more general transition matrix can be easily represented as

$$\boldsymbol{y}_n = \boldsymbol{\Phi} \boldsymbol{y}_0$$

where (7)

$$\boldsymbol{\Phi} = \boldsymbol{\mathcal{B}}_{n-1}\boldsymbol{\mathcal{B}}_{n-2} \dots \boldsymbol{\mathcal{B}}_1\boldsymbol{\mathcal{B}}_0.$$

The stability estimation of the system is now reduced to determining whether the modulus of eigenvalues of $\boldsymbol{\Phi}$ is less than one or not.

Until now the transition matrix $\boldsymbol{\Phi}$ was calculated from the known governing differential equations and dynamic behaviour of the system. However, the same can be calculated with the knowledge of state vectors $\boldsymbol{y}_n$ and $\boldsymbol{y}_0$. To be able to effectively solve the least square problem to estimate $\boldsymbol{\mathcal{B}}_n$ or $\boldsymbol{\Phi}$, the vector $\boldsymbol{y}_n$ or $\boldsymbol{y}_0$, respectively, needs to be converted to a matrix with linearly independent rows. This can be done by augmenting the state vector with additional state vectors defined within the same interval

$$\boldsymbol{Y}_{n+1} = \boldsymbol{\mathcal{B}}_n \boldsymbol{Y}_n \quad (8)$$

Right multiplying (8) by the pseudo-inverse $\boldsymbol{Y}_n^+$ results in,

$$\boldsymbol{\mathcal{B}}_n = \boldsymbol{Y}_{n+1} \boldsymbol{Y}_n^+ \quad (9)$$

Now the stability investigation can be carried out as usual by determining the modulus of spectrum of matrix $\boldsymbol{\mathcal{B}}_n$ or $\boldsymbol{\Phi}$.

3. Example

A time domain simulation is carried out for a micro-milling process using cutting forces in tangential F_t and radial F_r direction defined as:

$$F_t(t) = k_{tc} a_p h(t) \text{ and } F_r(t) = k_{rc} a_p h(t) \quad (10)$$

and instantaneous chip thickness $h(t)$ as:

$$h(t) = f_z \sin(\phi) + s(t-T) - s(t) \quad (11)$$

where, k_{tc} and k_{rc} are specific cutting coefficients, a_p is depth of cut, f_z is feed per tooth, s is the instantaneous radial tool displacement (see Figure 1), T is tooth passing period.

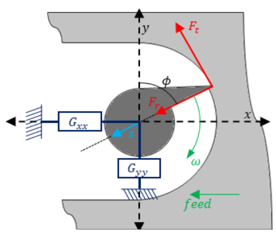

Figure 1. Milling geometry: cutting forces and tool displacement.

Based on the simulated parameters in [6], the simulation is advanced for a tool with 4 mm diameter and single flute with 45 degrees of flute angle, running at 30,000 rpm with 5 percent radial depth of cut with a feed of 150 µm/turn. Specific cutting coefficients are assumed to be $k_{tc} = 604$ Mpa and $k_{rc} = 220$ MPa. Symmetric dynamics are assumed with single mode at 720 Hz, stiffness 410 kN/m and a viscous damping ratio of 0.009.

Depth of cut is increased in steps of 0.24 mm, starting from 0.4 mm. For every depth of cut, the process is simulated for 200 turns and for every depth of cut transition matrix $\boldsymbol{\Phi}$ is calculated and its eigenvalues are calculated as well. Figure 2 Shows the modulus of the largest eigenvalue of $\boldsymbol{\Phi}$ plotted against simulated depth of cut.

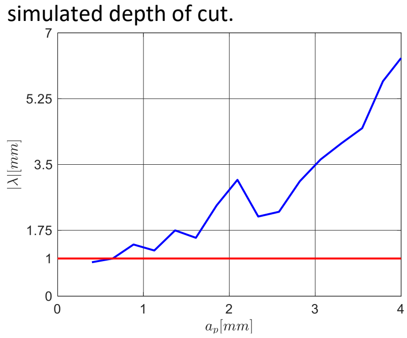

Figure 2. Modulus of eigenvelues against depth of cut

Poincare sections for the identified stable and unstable cutting parameters also confim the effectiveness of the described method (see Figure 3). The identified stability limit matches with the one identified by statistical method by the authors in [6].

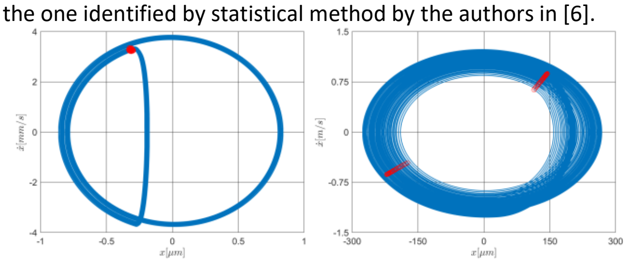

Figure 3. Poincare sections; left: $a_p = 0.4$ mm, stable; right: $a_p = 4$ mm, unstable.

4. Conclusion

This article presented a method to calculate the transition matrix of DDEs. This transition matrix is successfully used to investigate the stability of the micro-milling process. However, further investigation is required for real-time application of the presented indicator. Nonetheless, the effectiveness of the indicator is proven here using simulations. It is also expected to outperform the statistical methods to identify chatter.

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