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A method for sensor placement for high-precision position control of mechanical structures

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Abstract
This paper deals with high-precision position control of mechanical structures such as stages, optical elements or payload supports. The requirement of high precision positioning under vibration disturbances results in the demand of high control bandwidth. Elastic resonances of the control plant caused by its limited stiffness cannot be ignored, endanger the stability of the closed loop and hamper the performance. The techniques proposed in this paper support the achievement of high control bandwidth. Firstly, the design solution based on the use of tuned mass dampers and modelling of non-proportional damping is discussed. In the next step, oversensing, i.e. the use of additional sensors exceeding the number of degrees of freedom to be controlled, is considered. Due to the fact, that in many cases the solution based on damping is not sufficient alone and oversensing leads to additional costs of goods, optimisation of sensor placement including measuring direction is proposed as an additional powerful technique to achieve higher control bandwidth. The utilisation of different optimisation criteria including the observability of elastic modes of the plant taking into account the resolution capability of the measurement system resulting from the actual sensor configuration and preserving sensor noise amplification is analysed. Based on the plant model resulting from the finite-element approach, optimisation becomes a discrete multivariable task, which is not solved easily by conventional gradient-based methods. For this reason, it is proposed to use evolutionary algorithms for optimisation. Obtained results are demonstrated and discussed by means of examples. Finally, the presented techniques for increasing the control bandwidth are compared.

Keywords: sensor placement, position control, oversensing, modelling of non-proportional damping

1. Introduction

High position accuracy of mechanical structures under vibration disturbances can be achieved in many cases only due to application of control techniques and high control bandwidth. The limited stiffness of the mechanical structure and the actuator mechanism illustrated on Fig. 1 hamper the achievement of the higher control bandwidth due to resonances caused by elastic modes. The paper presents three approaches to handle this problem based on damping, oversensing and the optimisation of sensor placement.

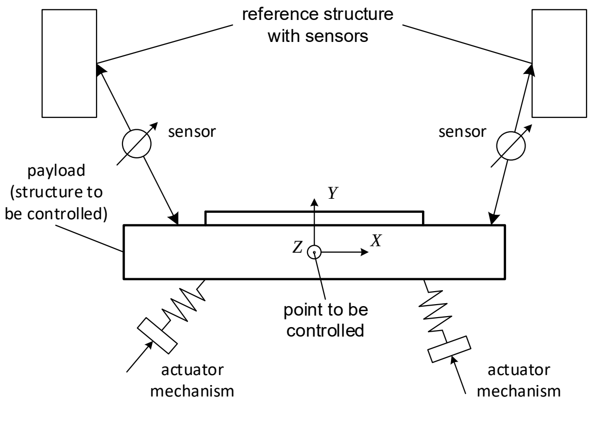

Figure 1. Payload to be controlled with respect to some translational and rotational degrees of freedom (only two actuators/sensors are shown)

The approach with damping can be considered as a passive one, while the last two approaches implying (active) feedback control. Fig. 2 shows the feedback control loop with a controller $\boldsymbol{C}$ plant $\boldsymbol{P}$ including the model of an elastic payload with an actuator mechanism and the transformation matrices $\boldsymbol{AK}$ and $\boldsymbol{MS}$. The control scheme depicted in Fig. 2 corresponds to the position control of a payload with respect to a certain point of interest (point to be controlled). The matrix $\boldsymbol{AK}$ describes the actuator kinematics. The measurement system matrix $\boldsymbol{MS}$ transforms the displacements on certain points of the payload structure to the motion of the point to be controlled. The both matrices $\boldsymbol{AK}$ and $\boldsymbol{MS}$ can be easily calculated based on the rigid body kinematics relations (e.g. Euler formulation).

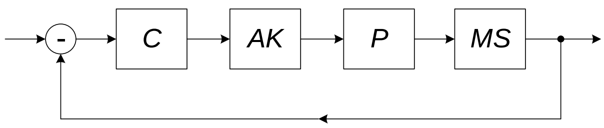

Figure 2. Feedback position control of an elastic payload with respect to a certain point of interest

As it will be considered below, the sensor placement and the modification of the measurement system matrix (in the oversensing case) are the crucial aspects for increasing the position accuracy of an elastic payload by means of higher control bandwidth. The reason for that is the destabilising effect of elastic modes, which can be minimised by optimisation. Additionally, the measurement system matrix has the big impact on the sensor noise propagation. This aspect, especially with a view to optimisation and the choice of the quality criterion, will be briefly discussed in this paper as well. The paper is organised as follows: firstly, the modelling and the damping approach will be presented, then – oversensing and optimisation of sensor placement followed by simulation results and conclusion.

2. Damping approach

One very powerful and pragmatic approach is to combine the optimisation of the sensor placement with the utilisation of damping in the mechanical structure, which can be realised in a mechanical design for example by tuned mass dampers. Due to the fact, that in this case the occurring damping is spatially localised, it cannot be modelled by a standard approach of the proportional damping (Rayleigh damping) with the assumption for the damping matrix $\boldsymbol{D}$ being a linear combination $\boldsymbol{D} = \alpha \boldsymbol{M} + \beta \boldsymbol{K}$ of the mass $\boldsymbol{M}$ and stiffness matrix $\boldsymbol{K}$ being symmetric and (semi-)positive definite [1, 2]. Instead of this, the following system description shall be considered

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{D}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{f}, \qquad (1)$$

where $\boldsymbol{x}$ and $\boldsymbol{f}$ are respectively the vector of generalised displacements (translations and rotations) and forces (point forces and torques). The model of the structure in form of (1) results as a rule from the discretisation by finite element method (FEM). On the other hand, for control tasks the state space representation allowing modal reduction is preferred. For the case of non-proportional damping, the crucial point is the normalisation of the eigenvectors. Unfortunately, the most popular FEM software tools provide the normalisation of the eigenvectors only with respect to the mass matrix $\boldsymbol{M}$ for the conservative case (without damping), so their utilisation for the construction of the state space model is not possible anymore (because in a general case the simultaneous diagonalization of three matrices is not possible). Instead of this, the following description is used

$$\begin{bmatrix} \boldsymbol{D} & \boldsymbol{M} \\ \boldsymbol{M} & \boldsymbol{\Theta} \end{bmatrix} \dot{\boldsymbol{X}} + \begin{bmatrix} \boldsymbol{K} & \boldsymbol{\Theta} \\ \boldsymbol{\Theta} & -\boldsymbol{M} \end{bmatrix} \boldsymbol{X} = \begin{bmatrix} \boldsymbol{f} \\ \boldsymbol{\Theta} \end{bmatrix}, \qquad (2)$$

where $\boldsymbol{X} \stackrel{\text{def}}{=} [\boldsymbol{x}\ \dot{\boldsymbol{x}}]^{\boldsymbol{T}}$ is the vector of the generalized displacements and velocities and $\boldsymbol{\Theta}$ are the zero vectors/matrices of the appropriate dimension. Thanks to this simple transformation the both matrices in (2) are symmetric, so the diagonalization with the complex eigenvectors and subsequent modal reduction become possible. To avoid the import of the complete mass matrix from a FEM software tools, the normalisation of the complex eigenvectors with respect to the extended matrix $\begin{bmatrix} \boldsymbol{D} & \boldsymbol{M} \\ \boldsymbol{M} & \boldsymbol{\Theta} \end{bmatrix}$ should be carried out within the framework of the FEM software (see [3]).

3. Oversensing approach

The oversensing approach allows to supress the observability of the crucial elastic modes by using more sensors than needed for measuring the only rigid body DOF R. Let $\boldsymbol{S}$ be an appropriate sensing matrix for measuring of some translational DOF with respect to a certain point of interest and some rotational DOF. Since the matrix $\boldsymbol{S}$ maps R rigid body DOF to the chosen amount of the sensor signals N, it possesses the full column rank (e.g. rank of six for the plant to be controlled over all spatial DOF) and the matrix $\boldsymbol{S}^{\boldsymbol{T}}\boldsymbol{S}$ is always non-singular, so the pseudoinverse for $\boldsymbol{S}$ can be always calculated as follows

$$\boldsymbol{MS_0} \stackrel{\text{def}}{=} [\boldsymbol{S^T S}]^{-1} \boldsymbol{S^T}, \qquad (3)$$

where we defined $\boldsymbol{MS_0}$ as the (initial) measurement system matrix mapping the sensor signals to the rigid body DOF of interest. The matrix $\boldsymbol{MS_0}$ has the full row rank of R, but its $N > R$ columns can only span the R-dimensional subspace ($\dim \mathrm{Col}\ \boldsymbol{MS_0} = R$), so the certain additional DOF $\chi \stackrel{\text{def}}{=} N - R$ can be used to affect the observability of the elastic modes without any impact to the measurement of the rigid body DOF. Moreover, in the modified measurement system matrix each of these χ degrees can be separately used for all rigid body DOF R, i.e. for each row of this matrix. It can be also interpreted, that some weighting factors of sensor signals could be now changed in a certain way without destroying the measurement of rigid body motion of interest. Indeed, following the rank theorem of linear algebra, the matrix $\boldsymbol{MS_0}$ has the χ-dimensional nullspace and in addition to the nominal case for the product of a sensing matrix by the corresponding measurement system matrix leading to the identity matrix $\boldsymbol{I}$ of dimension $R \times R$

$$\boldsymbol{MS_0 S} = [\boldsymbol{S^T S}]^{-1} \boldsymbol{S^T S} = \boldsymbol{I},$$

the same relation is also valid for the modified measurement system matrix

$$\boldsymbol{MS_1} \stackrel{\text{def}}{=} \boldsymbol{MS_0} + \begin{bmatrix} \alpha_{11} \dots \alpha_{1\chi} \\ \vdots \quad\quad \vdots \\ \alpha_{R1} \dots \alpha_{R\chi} \end{bmatrix} \begin{bmatrix} \boldsymbol{n}_1^T \\ \vdots \\ \boldsymbol{n}_\chi^T \end{bmatrix},$$

where $\boldsymbol{n}_1, \dots, \boldsymbol{n}_\chi \in \mathrm{Nul}\ \boldsymbol{MS_0}$ are the basis vectors spanning the null space of the matrix $\boldsymbol{MS_0}$ and α_{ij} are free parameters with $i = 1..R$ and $j = 1..\chi$. Indeed, each of the vectors $\boldsymbol{n}_1, \dots, \boldsymbol{n}_\chi$ is orthogonal to the row vectors of the matrix $\boldsymbol{MS_0}$ as vectors of the null space of $\boldsymbol{MS_0}$. On the other hand, due to (3) these vectors are orthogonal to the columns $\boldsymbol{s}_i$ of $\boldsymbol{S}$, i.e. the scalar product $(\boldsymbol{n}_j, \boldsymbol{s}_i) = \boldsymbol{n}_j^T \boldsymbol{s}_i = 0$ for $\forall i = 1..R$ and $\forall j = 1..\chi$, so it immedeately follows

$$\boldsymbol{MS_1 S} = \boldsymbol{MS_0 S} = \boldsymbol{I}.$$

The criterion for the choice of parameters α_{ij} shall be the minimisation of the observability of the crucial elastic mode(s). Besides the performance improvement due to higher achievable control bandwidth, the impact of sensor noise is to be taken into account as well. The utilisation of more sensors leads to the averaging effect and reduction of the sensor noise propagation, which can be easily characterised for a given measurement system matrix $\boldsymbol{MS}$ by the Frobenius norm $\|\boldsymbol{MS}\|_F = \sqrt{\sum_{i=1}^{R} \sum_{j=1}^{N} m_{ij}^2} = \sqrt{\mathrm{trace}[\boldsymbol{MS}^T \boldsymbol{MS}]}$ (on the assumption of the equal noise level of all sensors and the equal performance impact of all rigid body DOF of interest).

4. Optimisation of sensor placement

Three most important aspects of the optimisation of the sensor placement will be briefly described in this part: model preparation, optimisation criterion and the optimisation technique itself. In the first step, the model of the mechanical structure imported from the FEM shall be converted to a modal state space representation and parametrised by a node identification number. To extend the optimisation space we introduced virtual nodes with corresponding identification numbers, which differ from the original FEM nodes only due to the additionally defined rotated local coordinate systems allowing variation of the sensor measuring direction (achieved by orthogonal transformation of eigenvectors). Fig. 3 illustrates the approach and shows the “node clouds” on a payload structure being admissible for optimisation and a possible configuration of sensor measurement directions. As criterion we used the weighted sum of both: the observability Gramian of the crucial elastic modes and the Frobenius norm of the measurement system matrix to prevent high sensor noise amplification. Indeed, the reduced observability of the low-frequent elastic mode(s) allows increasing of the control bandwidth without penalty for the gain and phase margin, i.e. robustness in contrast to the notch-filters leading to the reduction of the phase margin and, as a consequence, to the higher maximum of the closed loop sensitivity function (sensitivity peaking).

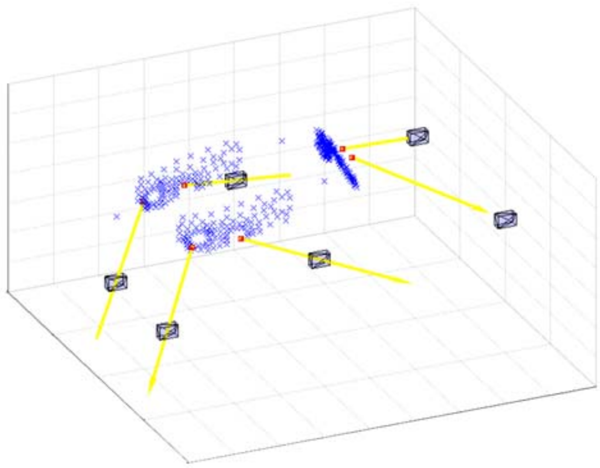

Figure 3. FEM nodes on a payload structure (blue crosses) and the sensor measurement directions (yellow lines), red boxes correspond to an initial configuration

Due to a discrete nature of the model resulting from FEM and, as a consequence, very high dimension of the optimisation space we propose to use for optimisation evolutionary algorithms [4], which provide satisfying results over a short period of calculation time, and depending on optional algorithm adjustments as population size and amount of generations.

5. Simulation results

As an example, an optical device as illustrated in Fig. 1 with an actuator resonance due to its limited stiffness was controlled over all spatial six DOF. In all cases we used the tuned mass dampers ensuring less sensitivity to uncertainties in the mechanical design (note: high-frequency resonances can be always filtered, e.g. by notch filters without big impact for phase margin and bandwidth). Figure 4 illustrates the simulation results by means of Bode and Nyquist plots for each DOF for three cases: a) with the initial sensor placement of six sensors and the crucial high resonance peak at the 1st elastic mode of the payload structure, which might limit robust behaviour; b) with an additional sensor (oversensing) providing the best suppression of 1st elastic mode but meaning additional costs of goods and c) optimisation of sensor placement by evolutionary algorithms providing a good trade-off between high bandwidth and sensor noise propagation without additional costs of goods.

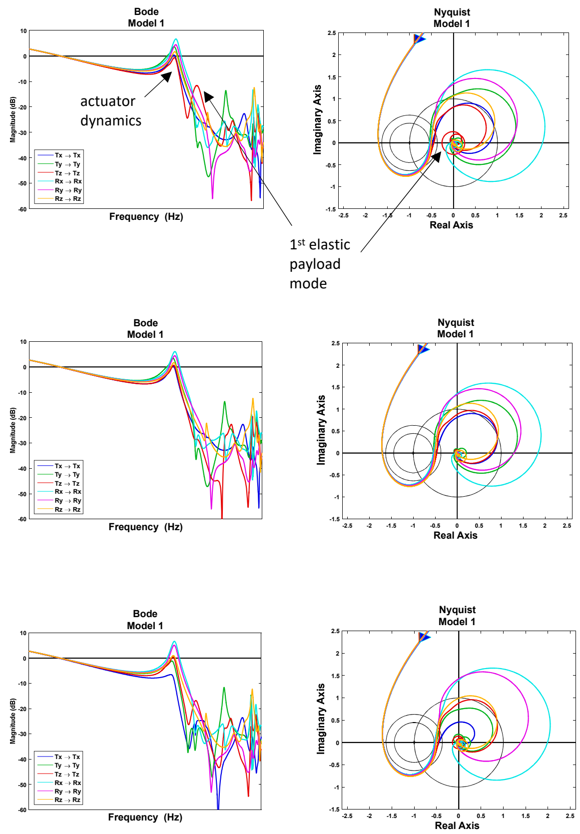

Figure 4. Open loop plots of a 6-DOF controlled payload: a) with 6 sensors – upper part; b) with 7 sensors (oversensing) – middle part and c) sensor placement optimisation by evolutionary algorithms – lower part

6. Summary and future work

Firstly, it is to note, that the obtained optimisation results can be hardly achieved by the manual seeking of the appropriate sensor placement or it typically requires an unacceptable time effort. Besides the good optimisation results compared to the initial case of the sensor placement, the question about the sensitivity/robustness of the obtained optimal case to small changes in the sensor placement/measuring direction remains open. The methods of the robustness analysis leading to the convex optimisation problems, which are very powerful from the calculation point of view, provide more or less conservative results and even if the non-robust constellation is possible, its probability is extremely low. Therefore, to analyse robustness, we propose to use some probabilistic approaches and/or to evaluate the sensitivity of the obtained optimal solutions by seeking for the worst-case sensor constellation in a certain neighbourhood of the optimal solution, for example again by the evolutionary algorithms. As a next research intention, the combination of the optimisation of both a plant to be controlled (as described above by means of sensor placement) and controller parameters as well, seems to be a very promising mechatronic approach.

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