

## Decentralized controller design in flexure-linked gantry by $H_2$ guaranteed cost optimization approach

Jun Ma<sup>1,2,3</sup>, Si-Lu Chen<sup>4</sup>, Wenyu Liang<sup>2</sup>, Chek Sing Teo<sup>3</sup>, Arthur Tay<sup>1,2</sup>, Abdullah Al Mamun<sup>1,2</sup>, Kok Kiong Tan<sup>1,2</sup>

<sup>1</sup> SIMTech-NUS Joint Lab on Precision Motion Systems, Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117582

<sup>2</sup> Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117583

<sup>3</sup> Agency for Science, Technology and Research, Singapore Institute of Manufacturing Technology, Singapore 138634

<sup>4</sup> Zhejiang Provincial Key Laboratory of Robotics and Intelligent Manufacturing Equipment Technology, Ningbo Institute of Industrial Technology, Chinese Academy of Sciences, Ningbo, Zhejiang, China 315201

ELEMJ@nus.edu.sg

### Abstract

Dual-drive H-gantry is widely used in many industrial processes that require high-speed and high-precision Cartesian motion. Unlike the rigid-linked gantry design, the flexure-linked design is able to prevent the damage of the cross-arm because a small degree of rotation angle is possible. However, the flexure-linked design may induce the resonant modes of the gantry because of high frequency control signals. To maintain the precision tracking of two carriages and minimize the chattering of control efforts, we aim to seek the most suitable flexure joints among the available ones while optimizing the decentralized feedback controller parameters under model uncertainties. In this way, good tracking performance is achieved without costly system re-design. In this work, we formulate the mechatronic design problem as an  $H_2$  guaranteed cost control problem. All the stabilizing feedback controller gains are parameterized over a convex set, and the global optimum is obtained by means of an outer-linearization-based algorithm. Experiments are conducted and the results successfully validate the effectiveness of the proposed design approach.

Keywords— optimal control, convex optimization, mechatronic design, flexure, parallel mechanism, dual-drive H-gantry

### 1. Introduction

In the configuration of a dual-drive H-gantry, each of the parallel carriage of the gantry stage is equipped with a linear actuator, which moves the cross-arm in tandem [1]. The cross-arm and the moving parts of actuators are linked by flexible joints to avoid high stresses at the fixation interfaces. Due to their particular mechanical configuration, the gantry has a very high precision over workspace ratio and high dynamic performances. Thus, it is widely used in industrial applications where high-speed high-precision Cartesian motion is required [2]. By imposing a pair of flexure joints in the gantry, the chance of total mechanical failure reduces and quick and cost-effective replacement is possible. However, such design may introduce resonant modes, making the end effector sensitive to high frequency control signals (chattering)[3]. Therefore, the tracking errors and the chattering of control efforts in two axes are to be minimized, where the flexure joints and motion controller need to be designed to meet the requirements.

### 2. Problem Formulation

The gantry setup is shown in Figure 1. The schematic diagram is shown in Figure 2(a), and it can be simplified as a two-mass system linked by a lightweight rod as in Figure 2(b). Notice that in our experiments, we only initiate our design based on the motion of Y-axis, while the X-axis moving carriage is held to be at the center position. Assume the parametric uncertainties exist in masses of two carriages, the gantry system can be modeled as a coupled linear uncertain system:

$$(M_1 + \Delta M_1)\ddot{y}_1 = K_f u_1 - \Gamma \dot{y}_1 + v - f_1 w(t), \quad (1a)$$

$$(M_2 + \Delta M_2)\ddot{y}_2 = K_f u_2 - \Gamma \dot{y}_2 - v - f_2 w(t), \quad (1b)$$

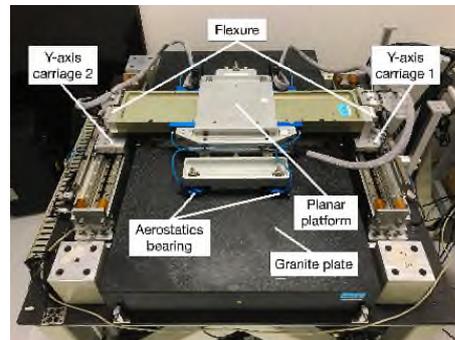


Figure 1. Setup of dual-drive H-gantry

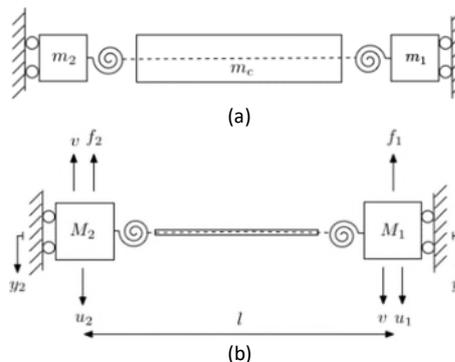


Figure 2. Schematic of dual-drive H-gantry (a) Actual model (b) Simplified model

with  $w$  assumed to be unit step function,  $M_1 = 16.5$  Kg,  $M_2 = 18.4$  Kg,  $f_1 = 0.1193$  N,  $f_2 = 0.1544$  N,  $K_f = 62.8$  N/A,  $\Gamma = 172.7$  Ns/m,  $v = K_v(y_2 - y_1)$  is the flexure force,  $K_v$  is the stiffness of the flexure. In this work, a 2-DOF control scheme consisting of feedforward control and feedback control is used, where  $u_1 = u_{1ff} + u_{1fb}$ ,  $u_2 = u_{2ff} + u_{2fb}$ .

An S-curve trajectory is used as the reference profile, where  $y_d = p_1$ ,  $\dot{p} = A_p p$ ,  $p^T = [p_1 \ p_2 \ p_3]$ ,  $A_p = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_{p1} & a_{p2} & a_{p3} \end{bmatrix}$ . (2)

To keep the precision tracking of two axes with minimizing the induced chattering from feedback control signals, the controlled output vector is defined as

$$z = W_z [e_1 \ e_2 \ \dot{u}_{1fb} \ \dot{u}_{2fb}], \quad (3)$$

with  $W_z = \text{diag}\{q_1, q_2, r_1, r_2\}$ ,  $e_1 = y_d - y_1$ ,  $e_2 = y_d - y_2$ , and  $q_1, q_2, r_1, r_2$  are weightings on  $e_1, e_2, \dot{u}_{1fb}, \dot{u}_{2fb}$ , respectively. The objective function is given by

$$J = \int_0^\infty z^T z dt. \quad (4)$$

Two pieces of flexure with different thickness (2mm and 3mm) are available for the experiments, where the stiffness of them are identified as  $K_{v,2mm} = 4887.3 \text{ N/m}$ ,  $K_{v,3mm} = 8693.7 \text{ N/m}$ . In this case, we aim to find the most suitable one and then determine the optimal feedback controller gains to minimize the cost function. We fix the flexure stiffness to the values of those two pieces first and group them into the dynamics of the gantry system. Next, we do the optimization for each case to obtain the optimal feedback controller and choose the set with the lowest upper bound to the cost in the presence of parametric uncertainties.

Based on the nominal model of the gantry system, we design the decentralized feedforward controller first, where

$$u_{1ff} = \frac{\Gamma}{K_f} \dot{y}_d + \frac{M_1}{K_f} \ddot{y}_d, u_{2ff} = \frac{\Gamma}{K_f} \dot{y}_d + \frac{M_2}{K_f} \ddot{y}_d. \quad (5)$$

Define  $x^T = [e_1 \ \dot{e}_1 \ \ddot{e}_1 \ e_2 \ \dot{e}_2 \ \ddot{e}_2 \ p_1 \ p_2 \ p_3]$ ,  $u_{fb}^T = [u_{1fb} \ u_{2fb}]$ , the augmented system is represented by the state-space model:

$$\dot{x} = (A + \Delta A)x + (B_2 + \Delta B_2)\dot{u}_{fb} + (B_1 + \Delta B_1)\dot{w}. \quad (6)$$

The feedback controller is given by

$$u_{fb} = -K \int_0^t x dt. \quad (7)$$

Here, we denote  $\text{blocdiag}$  as a diagonal constructor. To preserve the constraint from the decentralized nature of control architecture, the feedback control is given by

$$K = -\text{blocdiag}\{[k_{i1} \ k_{p1} \ k_{d1}], [k_{i2} \ k_{p2} \ k_{d2} \ 0 \ 0 \ 0]\}. \quad (8)$$

Equivalently, the controlled output  $z$  can be expressed as  $z = Cx + D\dot{u}_{fb}$ . (9)

The functional cost can be expressed as

$$J = \|H\|_2^2 = \text{Tr}[(C - DK)W_c(C - DK)^T] = \text{Tr}(B_1^T W_0 B_1), \quad (10)$$

where  $W_c$  and  $W_0$  are controllability and observability Gramians of the closed-loop system.

### 3. Proposed optimization algorithm

The feedback controller design problem in the presence of parametric uncertainties is converted to a convex optimization problem, we can use an outer-linearization technique presented in [4] to solve the problem, which sequentially generates separating cutting planes. In this case, the optimal solution is calculated by a convergent sequence of linear programming.

We set the all of the eigenvalues of  $A_p = -4$  to obtain an S-curve profile, thus,  $a_{p1} = -64$ ,  $a_{p2} = -48$ ,  $a_{p3} = -12$ . Two carriages of the gantry are targeted to move  $-0.1\text{m}$  along the Y-axis. Notice that the optimization results are applicable if the S-curve profile is shifted or mirror flipped up and down. The reference profile is illustrated in Figure 3.

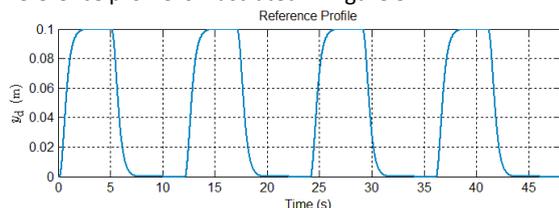


Figure 3. Reference profile

The weightings are set to  $q_1 = q_2 = 1, r_1 = r_2 = 1$ . We assume the uncertainties  $\Delta M_1$  and  $\Delta M_2$  are within  $\pm 10\%$  range of  $M_1$  and  $M_2$ . When the 2-mm flexure is used, the stopping criterion of the optimization algorithm is met after 608 iterations, the optimal controller gain is given by

$$K = -\text{blocdiag}\{[18.5 \ 216.5 \ 1.2], [0.5 \ 196.3 \ 0.3 \ 0 \ 0 \ 0]\},$$

and the upper bound to the cost is given by  $1.73 \times 10^{-5}$ . Next, we use the 3-mm flexure to do the optimization. After 361 iterations, the optimal controller is given by

$$K = -\text{blocdiag}\{[126.2 \ 2340.0 \ 14.2], [91.0 \ 909.8 \ 9.0 \ 0 \ 0 \ 0]\},$$

and the upper bound is given by  $9.09 \times 10^{-6}$ . Compare 2-mm and 3-mm flexure joints, the latter one results in a lower upper bound to the cost. Thus, we choose the 3-mm flexure for the experiments. Tracking errors, control efforts and their chattering of two axes are illustrated in Figure 4.

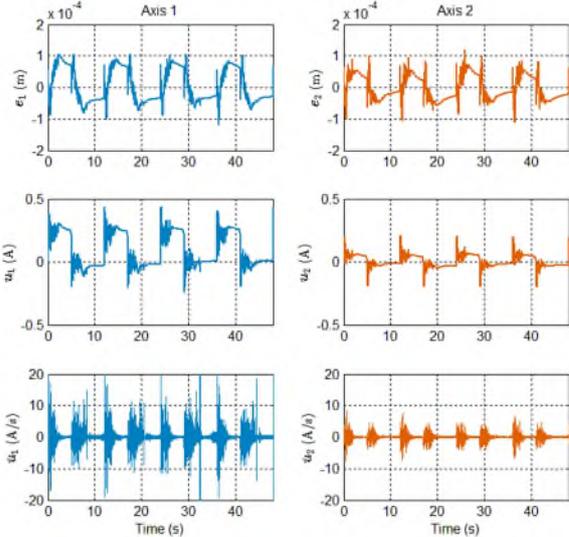


Figure 4. Tracking errors, control inputs and their chattering of two axes

### 4. Conclusion

In this work, we aim to maintain good tracking and minimized induced vibration to the end-effector plane for a flexure-linked dual-drive H-gantry, where the most suitable flexure and the optimal decentralized feedback controller are to be designed. By fixing the stiffness of provided flexure to certain values, we aim to optimize the decentralized feedback controllers and choose the set of flexure and controllers with the lowest cost. The mechatronic design problem is considered as an  $H_2$  guaranteed cost control problem and the optimal gain is obtained using an outer-linearization-based algorithm. As validated by experiments, the 2-DOF control scheme is capable to give good performance in terms of the tracking accuracy and the chattering of control efforts.

### References

- [1] Hu C, Hu Z, Zhu Y, et al. Advanced GTCF-LARC Contouring Motion Controller Design for an Industrial X-Y Linear Motor Stage With Experimental Investigation[J]. *IEEE Transactions on Industrial Electronics*, 2017, **64**(4): 3308-3318.
- [2] Li D, Yoon S W. PCB assembly optimization in a single gantry high-speed rotary-head collect-and-place machine[J]. *The International Journal of Advanced Manufacturing Technology*, 2017, **88**(9-12): 2819-2834.
- [3] Ma J, Chen S L, Kamaldin N, et al. A novel constrained  $H_2$  optimization algorithm for mechatronics design in flexure-linked biaxial gantry[J]. *ISA transactions*, 2017, **71**: 467-479.
- [4] Geromel J C, Bernussou J, Peres P L D. Decentralized control through parameter space optimization[J]. *Automatica*, 1994, **30**(10): 1565-1578.