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On precise modelling of very thin flexure hinges

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Abstract

The continuously rising demands for precision favour the application of monolithic compliant mechanisms with flexure hinges in various fields of precision engineering. It gives way to an almost frictionless and precise motion even under vacuum conditions. These advantages have made compliant mechanism an integral component of high precision weighing cells. A downside of compliant joints is their stiffness towards deflection, which limits the sensitivity of the overall system. Consequently, the flexure hinges are manufactured as thin as possible. The present limit in terms of manufacturing technology is within the range of 50 µm. The objective of predicting the behaviour of highest precision weighing cells by modelling is directly interconnected with the exact knowledge of the behaviour of flexure hinges in terms of stiffness. Especially, for flexure hinges with high aspect ratios, typically found in weighing cells, the existing analytical equations and finite element models show pronounced deviations. The present research effort is dedicated to a clarification of this observation. Structure mechanical finite element models are developed to identify the deviations of the models precisely. Results obtained are compared to analytical results and conclusions for the modelling of thin flexure hinges are drawn.

Keywords: compliant mechanism, semi-circular flexure hinge, stiffness, restoring force, finite element method

1. Introduction

High precision applications often require features that cannot be achieved through conventional design. Instruments are designed to obtain measurements as precise and repeatable as possible with very high resolution. Compliant mechanisms have proven to be suitable for these systems due to their almost frictionless and precise motion. For some applications, the stiffness of the flexure hinges represents one of the important properties that needs to be known precisely [1].

Many authors have proposed expressions to determine the stiffness components of various flexure hinges [2-6], especially their rotational stiffness. However, these values differ when compared to finite element (FE) calculations. This fact was evidenced in the works of Smith [3] and Yong [4] where analytical and approximate equations for semi-circular flexure hinges were compared to 2D-FE models, showing an increasing relative deviation when decreasing the minimal height-radius h/R ratio. This ratio was used to classify thin, intermediate and thick hinges in [5]. The hinge geometries, treated here, belong to the lower range limit for thin flexure hinges.

In the design of flexures, out-of-plane compliances are diminished by increasing the width of the hinge to avoid parasitic motions. In the case of thin hinges, this produces very high aspect ratios *b/h* for which the assumptions made in analytical expressions and 2D-FE models are no longer valid and high deviations to 3D-FE models can be observed. This study seeks to clarify the mentioned phenomenon. The remaining sections are as follows. In Section 2, the modelling procedure of the flexure hinge used in this work is presented. In Section 3, the results of the FE model are compared with the analytical models and 2D-FE models. Finally, the deviations observed are discussed in Section 4.

2. Finite element modelling of thin flexure hinges

Due to its widespread use in high precision systems and availability of models, a semi-circular flexure hinge according to Fig. 1 is studied:

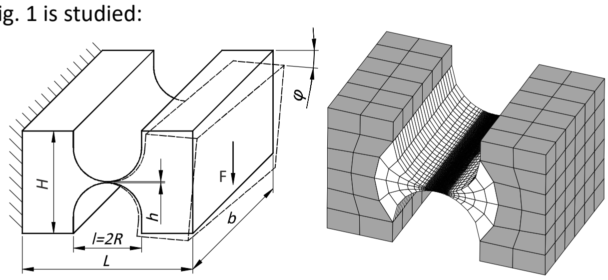

Figure 1. Geometric parameters and FE model of the flexure hinge

FE modelling of a notch flexure hinge is suitable for mapped meshing because of its simple geometry. Due to high aspect ratios of the hinge, the number of elements can be quite high or they can have undesirably high aspect ratios. Thus, the meshing strategy shown in Fig. 1 will be used. The domain is divided in zones: a highly refined central zone, a transition zone and a rough outer zone. Element size in the out-of-plane direction depends on the minimum element size of each zone and a maximum aspect ratio. These unmatching meshes are connected through multipoint constraints. The number of elements amounts to 18912. The simulations were conducted in ANSYS Mechanical using 20-node brick elements, SOLID186, and geometric nonlinearities were considered. The parameters of the model are shown in Tab. 1.

The rotational stiffness is determined with the bending moment and the rotation angle as $k_\varphi = M/\varphi$, where M is generated by an applied force F multiplied by the lever arm $L/2$ and φ is calculated from the displacement of two points of the free end. The bending moment M was the same for all simulations so that it produces rotations of less than 1° in all hinges.

Table 1. Parameters of the model

Total length	L	15 mm	**Elastic modulus**	E	71 GPa
Total height	H	9 mm			
Hinge length	l	6 mm	**Poisson's ratio**	ν	0.33
Min. height	h	50 µm			
Width	b	10 mm	**Density**	ρ	2.7 g/cm^3

3. Analytical vs finite element models

Most analytical expressions found in literature are derived from the bending differential equation of Euler-Bernoulli's beam theory [2-4]. This theory assumes that cross sections remain planar after deformation, which is only valid for long slender beams [2]. This was also assumed in the nonlinear beam theory, used for deriving the design equations by Linß [6]. Another analytical approach was presented by Tseytlin [5], where equations were approximated from the plane differential equation of continuum mechanics, but considering the modified elastic modulus $E' = E/(1-\upsilon^2)$, which applies for thin films and wide plates [5].

A comparison of the relative deviation of stiffness between the 3D-FE model and analytical expressions aforementioned is shown in Fig. 2. 2D-FE models using the same mesh strategy as the 3D model were generated and are also compared. One of the 2D models was configured for a plane stress state and the other one for a plane strain state. The comparison is made by increasing the minimal height h while maintaining all other parameters constant. As for the FE models, loading conditions and meshes were also kept constant.

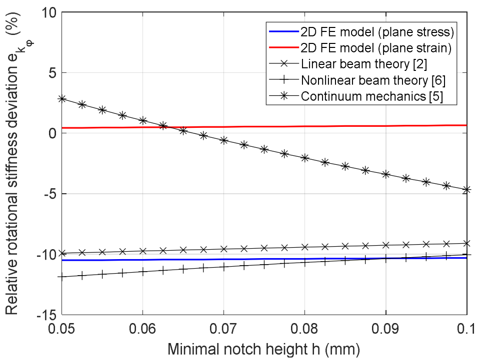

Figure 2. Percentage deviations of k_φ compared to 3D-FE model.

Two observations can be done from Fig. 2. First, the results from Euler-Bernoulli's beam theory and nonlinear beam theory, as well as the 2D plane stress model, deviate in great manner (between 9% and 12%) from the 3D-FE model with increasing deviation as the minimal thickness h decreases. Second, the least deviations are obtained from the 2D plane strain model, which assumes that the out-of-plane dimension is very large [7]. These two facts give rise to the assumption that there is a dependency on the aspect ratio b/h in the deviations.

All aforementioned expressions consider a linear relationship between the stiffness and the width of the hinge. If this were true, a graph between a normalized stiffness k_φ/Eb and a changing minimal height should not vary for FE calculations of the 3D model using different widths. However, this is not the case as shown in Fig. 3, where the normalized stiffness was calculated for increasing minimal height for different widths.

Fig. 3 shows that with increasing width b, thus, increasing aspect ratio b/h, the behaviour changes from a state of plane stress to a state of plane strain, which also considers a linear relationship between the stiffness and the width. An important observation is that for a certain value of the width, the results converge into a state of plane strain. This is discussed in Section 4.

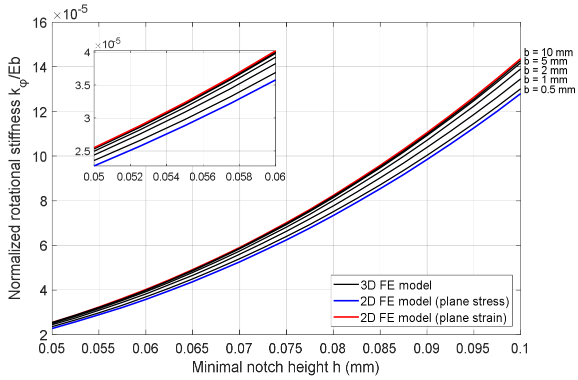

Figure 3. Normalized stiffness k_φ/Eb for changing width b.

4. Discussion of results

A nonlinear relationship between the stiffness and the width, with plane stress and plane strain as boundary cases, can be identified in Fig. 3. Beam theory assumes a plane stress state, which can be seen in the proximity of the deviations from analytical and 2D-FE plane stress models in Fig. 2 and in [4], but the modelling assumtions for plane stress are only valid for small out-of-plane dimensions [7], thus, for low b/h ratios. If this ratio is very large, strains in the out-of-plane direction tend to zero [7] (plane strain). As a result, the stress in this direction increases, reducing the strain in the longitudinal direction. The stiffness increases. The observed convergence of the curves into the 2D plane strain model with increasing width in Fig. 3 occurs because the out-of-plane stress tends to a constant value over the width. The simulations confirm this behaviour. Only if the ratio b/h is falling short of a certain value, an approximately linear relation between stiffness and width can be assumed for a discrete height h. The deviation of the 3D model to the plane strain model shown in Fig. 2 exists because the out-of-plane stress is not constant as assumed in the model but reduces to zero at the outer boundaries of the flexure hinge. It is important to remark that the continuum mechanics model shows less deviation for the thinnest hinges as it includes the term $1/(1-\upsilon^2)$, which differentiates the flexural rigidity of beams and plates [7].

From these observations, it can be concluded that emphasis should be placed on the selection of suitable modelling assumptions when calculating the behaviour of thin flexure hinges with high b/h ratios.

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