

## Investigations of the geometrical scaling in the systematic synthesis of compliant mechanisms

Philipp Gräser<sup>1</sup>, Sebastian Linß<sup>2</sup>, Thomas Räder<sup>1</sup>, Lena Zentner<sup>2</sup>, René Theska<sup>1</sup>

Technische Universität Ilmenau, Department of Mechanical Engineering

<sup>1</sup>Institute of Design and Precision Engineering, Precision Engineering Group

<sup>2</sup>Compliant Systems Group

E-Mail: [philipp.graeser@tu-ilmenau.de](mailto:philipp.graeser@tu-ilmenau.de)

### Abstract

Due to their advantages to conventional mechanisms, compliant mechanisms with flexure hinges are becoming increasingly important in precision engineering and micro technology. A systematic synthesis of compliant mechanisms allowing goal-oriented solutions in the design process of guiding and motion transmission systems in the field of ultra-high precision is missing. Towards this, this paper presents investigations of the influence of the geometrical scaling on the functional parameters of compliant mechanisms. Deviations in the results derived by analytical or numerical models and measurements done at manufactured prototypes are discussed. For comparison a previously investigated parallel four-bar linkage is chosen. At first investigations based on the rigid-body model are done to derive equations for the parameters of the path of motion (e.g. displacement, straight-line deviation) in dependence of the geometrical scaling factor. Subsequent FEM simulations for selected mechanisms with varying scaling factors are done. To verify the simulations two prototypes are measured. In an evaluation process the difference between the FEM and measurement results are shown and potential causes are discussed. It is shown, that the method of geometrical scaling is valuable in the systematic synthesis of compliant mechanisms in precision engineering.

Keywords: Compliant mechanism, geometrical scaling, flexure hinge, polynomial contour

### 1. Introduction

In a large field of applications, compliant mechanisms [1] with notch flexure hinges [2] are the base of motion systems [3]. Thus, simplified and time-saving approaches for the systematic synthesis of these mechanisms are of interest. This contribution is focused on the influence of uniform geometric scaling of those mechanisms on the functional parameters. Analytical calculations, FEM-based simulations and measurement taken at manufactured prototypes are carried out and discussed. As the conclusion, it becomes clear, that uniform geometric scaling can be used to adapt mechanisms to desired applications without the need to rerun simulations.

### 2. Synthesis

Geometrical scaling is done for a four-bar linkage with known kinematic behaviour. The design guidelines for the synthesis of this mechanism have been taken from previous investigations. In Figure 1a the geometrical parameters of the rigid-body model are shown. They are optimized for a large stroke of motion of the coupler point C respectively C' (Fig. 1) and a small straight-line deviation  $v_y$  [4].

To transfer the rigid-body model into a compliant mechanism all idealized revolute joints are replaced by flexure hinges. The rotation axes of the hinges remain in the same positions in the none-deflected mechanism, as shown in Figure 1b. The geometrical parameters of the flexure hinges are already optimized based on the relative rotation angles of the hinges, which ensure a sustainable strain  $\epsilon$ . The strain in the mechanism is aimed to be constant for any investigated scaling factor. Thus, the input displacement is scaled too, which leads to the same hinge rotation angles. The notch contour of the

hinges follows a polynomial function of 6<sup>th</sup> order. The minimum notch height is chosen as 300  $\mu\text{m}$  based on previous investigations [5].

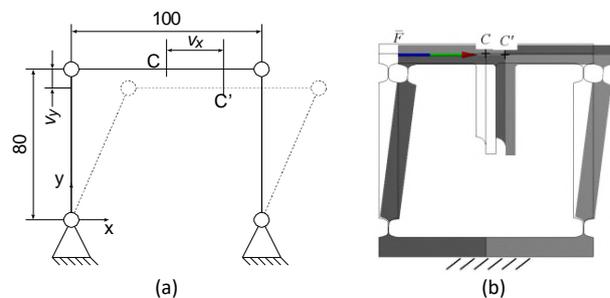


Figure 1. Synthesis approach: replacement of the revolute joints in the rigid-body model of a parallel four-bar linkage with  $s = 1$  (a) by flexure hinges with polynomial hinge contours in the compliant mechanism (b)

### 3. Geometrical scaling

Starting from a compliant mechanism optimized for a stroke of  $\pm 10$  mm, variants with  $\pm 1$  mm up to  $\pm 20$  mm with different outer dimensions of the compliant mechanisms are derived by up and down scaling. All geometric parameters are scaled with the same factor  $s$ . Three further examples ( $s = 0.1, 0.5, 2$ ) are investigated by analytical calculations and FEM-based simulations. Detailed FEM simulations are necessary to show differences in the path of motion due to a varying hinge contour in subsequent investigations and to verify the previous analytical analysis. The derived output parameters are compared for the initial and the scaled mechanisms, see Table 1. Most interesting parameters are the displacements  $v_x$  and  $v_y$ , the rotation angle of the coupler  $\delta$  (the output element), the maximum VON MISES strain  $\epsilon_{\text{max}}$ , the input force  $F$  and the first

eigenfrequency  $f_E$ . The rotation of the coupler  $\delta$  results from the moment induced by the input and the moment out of the force reactions in the material coherent pairings. Also a deformation of the output element is observed. For the initial model the mesh for the FEM simulation is optimized and maintained for all other models. In each simulation large deflections are considered.

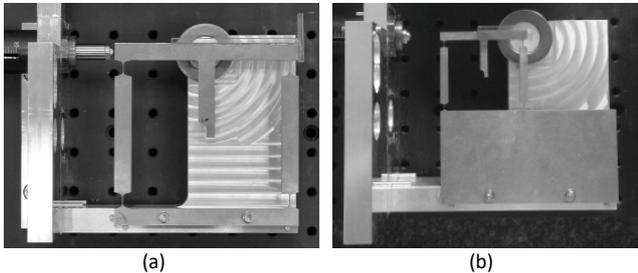
**Table 1.** Results of FEM-based simulations of the elasto-kinematic behaviour of a compliant parallel four-bar linkage in dependence of the geometrical scaling factor  $s$

	$s = 0.1$	$s = 0.5$	$s = 1$	$s = 2$
displacement $v_x$ in mm	1.0	5.0	10.0	20.0
displacement $v_y$ in mm	-0.061	-0.307	-0.614	-1.23
$\delta$ in arc sec	0.29	0.29	0.29	0.29
$\epsilon_{\max}$ in %	0.48	0.48	0.48	0.48
$F$ in N	0.015	0.365	1.461	5.844
eigenfrequency $f_E$ in Hz	104.04	20.84	10.42	5.21

It can be seen, that the displacements scale directly and the eigenfrequency scales reciprocally proportional with the scaling factor  $s$ . The input force is in a quadratic dependence. The rotation angle of the coupler and the maximum strain are both not influenced by geometrical scaling.

#### 4. Prototypes and measurement

The FEM simulation results are verified by the measurement of two manufactured prototypes. Figure 2a shows the initial mechanism. The second prototype, scaled with the factor  $s = 0.5$ , is shown in Figure 2b. The prototypes are manufactured of the aluminium alloy EW 7075 by wire electrical discharge machining.



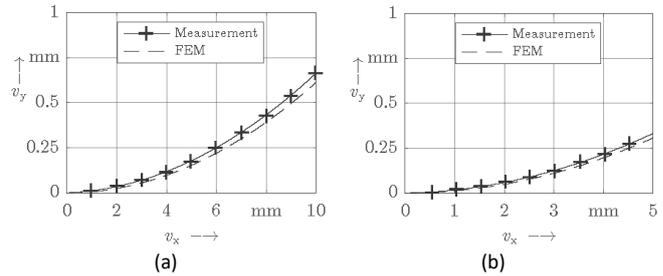
**Figure 2.** Manufactured prototypes in the test bench with a linear actuator at the left side of the coupler and two planar, orthogonal arranged mirrors for interferometric measurement: the initial parallel four-bar linkage with  $s = 1$  (a); the scaled mechanism with  $s = 0.5$  (b)

The test bench is an interferometric length measurement based setup, created for previous investigations. It allows the simultaneous and direct measurement of the displacements in  $x$  and  $y$  direction  $v_x, v_y$  with nanometre precision. The rotation angle of the coupler  $\delta$  is calculated based on the taken measurements [4]. Motion input is initiated by a linear actuator with a resolution of  $0.1 \mu\text{m}$ . To compensate the dead weight a rolling ball below the coupler is used. With a modified setup it is also possible to measure the input force  $F$ , not shown here. Up to now, there is no possibility, in the presented setup, for the exact measurement of the maximum strain and the first eigenfrequency.

#### 5. Discussion of results

In general, the comparison of the simulation and the measurement results are in good correlation for all output values. As an example, the displacement  $v_y$  in dependence of

the input displacement  $v_x$  is shown for the simulation and the measurement in Figure 3. Comparing Figure 3a with 3b, it is obvious, that the displacement  $v_y$ , which describes the straight-line deviation, changes with the scaling factor  $s$ , as expected based on the simulation results.



**Figure 3.** Results for the displacement of the output element of the FEM simulations and the measurement of prototypes of the compliant mechanisms with the geometrical scaling factor  $s = 1$  (a) and  $s = 0.5$  (b)

Thus the measurements taken at the two prototypes confirm the simulation results. Occurring deviations are mainly a result of manufacturing tolerances of the prototypes and varying disturbances on the setup of the test bench.

#### 6. Conclusion

The presented investigations show the influence of geometrical scaling of compliant mechanisms on their elasto-kinematic behaviour. The influence on functional parameters, like the displacement, rotation angle, maximum strain value, input force and eigenfrequency is determined by FEM-based simulations. The results of the simulations are verified by the exemplary measurement of prototypes.

It is shown, that compliant mechanisms can be varied by geometrical scaling without rerunning of simulation. Thus, based on the analytic calculations of the output parameters of a known initial model, geometrical scaling is a time-saving and sufficient way to vary a compliant system to given functional requirements. Accordingly, geometrical scaling of the mechanism is a constructive and efficient method in the systematic synthesis of compliant mechanisms.

#### Acknowledgments

The development of this project is supported by the Deutsche Forschungsgemeinschaft (DFG) under Grant No. TH 845/5-2 and ZE 714/10-2.

#### References

- [1] Howell L L, Magleby S P, Olsen B M: Handbook of Compliant Mechanisms, Wiley, Chichester, 2013.
- [2] Tseytlin Y M: Notch flexure hinges: An effective theory, Rev. Sci. Instrum., 73, 3363–3368, 2002.
- [3] Luo Y, Liu W, Wu L: Analysis of the displacement of lumped compliant parallel-guiding mechanism considering parasitic rotation and deflection on the guiding plate and rigid beams. Mech. Mach. Theory 91, 50-68, 2015.
- [4] Gräser P, Linß S, Zentner L, Theska R: Design and experimental characterization of a flexure hinge-based parallel four-bar mechanism for precision guides, in: Microactuators and Micromechanisms, Springer International Publishing, Cham, 139-152. 2017.
- [5] Linß S, Milojevic A, Pavlovic N D, Zentner L: Synthesis of Compliant Mechanisms based on Goal-Oriented Design Guidelines for Prismatic Flexure Hinges with Polynomial Contours, in: 14th IFToMM World Congress, Taipei, Taiwan, 2015.