euspen's 18th International Conference & Exhibition, Venice, IT, June 2018
www.euspen.eu

Deriving the mathematical principle of the shaft runout detected by SelfA+ and confirming it

Yuri Ueyama[1], Ryoshu Furutani[1], Tsukasa Watanabe[2]

[1]*Department of Mechanical Engineering, Graduate School of Engineering, Tokyo Denki University*
[2]*Dimensional Standards Group, Research Institute for Engineering Measurement, National Institute of Advanced Industrial Science and Technology*

yuri.ueyama@gmail.com

Abstract

We derived the mathematical principle about the shaft runout detected by SelfA+ which is developed by National Institute of Advanced Industrial Science and Technology (AIST) in Japan and confirmed it. SelfA+ is developed by adding the function to detect the shaft runout which is one of failure factors of machine tools to the self-calibration rotary encoder called SelfA (Self-Calibratable Angle Device) that can detect an angle error by arranging the plural sensors with equal angular intervals. Therefore, it is expected that SelfA+ will be used as a sensor to predict the failure of machine tools. In our previous study, we developed the experimental device to generate a pseudo shaft runout by moving the encoder housing on the sensor side instead of moving the rotational axis directly. As a method of moving the encoder housing, the mechanism that the shaft runout was generated by the XY piezo stage was induced on the encoder housing. In this way, it was possible to quantitatively evaluate the amount of shaft runout detected by SelfA+. In order to accurately evaluate the shaft runout detected by SelfA, in this time we carried out the experiment to compare the shaft runout evaluated by SelfA+ with the shaft runout quantitatively generated by the XY piezo stage. As the result, it is proved that SelfA+ can detect the shaft runout without the specific frequency components of the shaft runout to depend on the number of arranged sensors. Therefore, in order to analyze the lack of the specific frequency components of the shaft runout, we expand the mathematical algorism of SelfA+ and could successfully derive relational expressions that can support our experimental results.

Keywords Rotary encoder Runout Measurement instrument Angle

1. Introduction

Since the shaft runout causes the mechanical failure, damage and decreases in the precision of a machine having a rotating mechanism, the periodic inspection of the shaft runout is important in terms of safety and life management of the machine. The high accurate measuring devices such as a laser displacement gauge, a capacitance sensor and so on are used for measuring the shaft runout [1]. However, these equipments are not practical for having problems such as an increase in size and cost. Therefore, if a rotary encoder incorporated as a sensor for measuring the angle positions of the rotating mechanism can measure the shaft runout, it is possible to realize the measuring device of the shaft runout overcomes these problems.

In order to evaluate the ability of SelfA+ which can detect a shaft runout, we need the experiment system to generate the shaft runout quantitatively to a rotating shaft. In our previous study, we developed the experiment system having the mechanism to generate the pseudo shaft runout by moving the sensor units of the rotary encoder instead of moving the rotational shaft directly and evaluated this system [2,3].

In this paper, we derived the mathematical principle quantitatively for SelfA+ from SelfA and compared the amount of the shaft runout detected by SelfA+ with the value of the mathematical model.

2. Mathematical principle of SelfA+

Angle error factors detected by a rotary encoder are generally divied into a periodic rotational error and a translational direction error like the following.

・A periodic rotational error is described like sinusoidal wave and includes the scale lines error and the eccentricity error.

・A translational direction error is the shaft runout due to a radial direction of a rotational shaft.

SelfA can evaluate these angle error factors. Also, SelfA+ is the method to separate only a shaft runout from the angle error factors detected by SelfA.

2.1. Angle error factors detected by SelfA

SelfA is based on the principle of self-calibration composed by arranging the sensor of M units with equal angular intervals around the scale line number of N in the scale plane [5,6]. The angle error $\delta_{m,\theta}$ detected by sensor of $m(=0,1,\ldots,M-1)$ unit in the angular position θ for one rotation is defined as equation (1), which includes the periodic rotational error $a_{m,\theta}$ and the translational direction error $r_{m,\theta}$.

The periodic rotational error $a_{m,\theta}$ and the translational direction error $r_{m,\theta}$ can be defined the following reason. The angle error $\delta_{m,\theta}$ can be expressed the complex discrete fourier transform to become the sum of the periodic function as angle has the characteristics to return to the original position at one

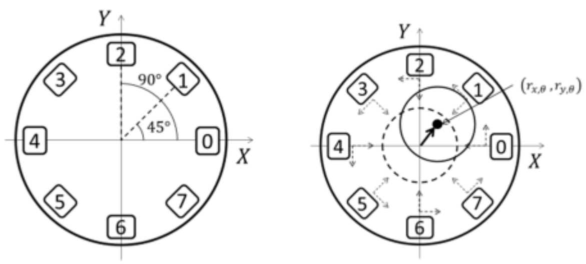

Figure 1. periodic rotational error **Figure 2.** translational direction error

rotation. Therefore, $a_{m,\theta}$ is defined as equation (2) since the angle error detected by sensor of m unit is shifted by each angular positions $\theta_m \left(= \frac{2\pi}{M} m\right)$ where sensor of M units is arranged as shown in Figure 1. Then, $r_{m,\theta}$ is defined as equation (3) by applying the projective matrix to the coordinate system for each sensor units using X and Y direction of the translational direction components $(r_{x,\theta}, r_{y,\theta})$ as shown in Figure 2.

$$\delta_{m,\theta} = a_{m,\theta} + r_{m,\theta} \tag{1}$$

$$a_{m,\theta} = \frac{1}{N} \sum_{k=0}^{N-1} f_k e^{-ik(\theta+\theta_m)} \tag{2}$$

$$r_{m,\theta} = \cos\left(\theta_m + \frac{\pi}{2}\right) \times r_{x,\theta} + \sin\left(\theta_m + \frac{\pi}{2}\right) \times r_{y,\theta} \tag{3}$$

$$\begin{cases} r_{x,\theta} = \dfrac{1}{N} \displaystyle\sum_{k=0}^{N-1} g_{x,k} e^{-ik\theta} \\ r_{y,\theta} = \dfrac{1}{N} \displaystyle\sum_{k=0}^{N-1} g_{y,k} e^{-ik\theta} \end{cases}$$

2.2. Priciple of SelfA⁺

At first, in order to separate only the translational direction error from the angle error detected by each sensor units as shown in the equation (1), the phase of the fourier components in the equation (1) is shifted by each angular positions $(= \theta_m)$, the equation (4) is obtained. Next, take the average of the sensor unit number M at the equation (4) as the equation (5). Therefore, the difference of the equation (4) and the equation (5) removes the periodic rotational error $a_{m,\theta}$ as shown in the equation (6). Finally, by returning the shifted each angular positions$(= \theta_m)$, the equation to detect only the translational direction error by each sensor units without the specific fourier components is obtained.

$$\begin{aligned} \delta_{m,\theta-\theta_m} &= a_{m,\theta-\theta_m} + r_{m,\theta-\theta_m} \\ &= \frac{1}{N} \sum_{k=0}^{N-1} \{f_k + p_{m,k} g_{x,k} + q_{m,k} g_{y,k}\} e^{-ik\theta} \end{aligned} \tag{4}$$

$$\begin{cases} p_{m,k} = \dfrac{e^{i(k+1)\theta_m} e^{i\frac{\pi}{2}} + e^{i(k-1)\theta_m} e^{-i\frac{\pi}{2}}}{2} \\ q_{m,k} = \dfrac{e^{i(k+1)\theta_m} e^{i\frac{\pi}{2}} - e^{i(k-1)\theta_m} e^{-i\frac{\pi}{2}}}{2i} \end{cases}$$

$$\begin{aligned} \overline{\delta_{\theta-\theta_m}} &= \frac{1}{M} \sum_{m=0}^{M-1} \delta_{m,\theta-\theta_m} \\ &= \frac{1}{N} \sum_{k=0}^{N-1} \left\{ f_k + \frac{1}{M} \sum_{m=0}^{M-1} \left(p_{m,k} g_{x,k} + q_{m,k} g_{y,k} \right) \right\} e^{-ik\theta} \end{aligned} \tag{5}$$

$$\delta_{m,\theta-\theta_m} - \overline{\delta_{\theta-\theta_m}} = \frac{1}{N} \sum_{k=0}^{N-1} z_{m,k}\, e^{-ik\theta}$$

$$\begin{aligned} z_{m,k} = &\left(p_{m,k} - \frac{1}{M} \sum_{m=0}^{M-1} p_{m,k} \right) g_{x,k} \\ &+ \left(q_{m,k} - \frac{1}{M} \sum_{m=0}^{M-1} q_{m,k} \right) g_{y,k} \end{aligned} \tag{6}$$

3. Experiment

The experiment is conducted under the following conditions. The amplitude of the shaft runout in the X and Y directions are 1 [μm], and the phase in the X direction is $\frac{\pi}{2}$ [rad], the phase in the Y direction is π [rad]. The period of the generated shaft runout in one rotation is 18 kinds of 1 to 18. Moreover, as the condition of the rotary encoder, type is optical, the scale lines $N = 18000$ and sensor of $M = 8$ units.

4. Result

The shaft runout generated under this condition $(g_{x,k}, g_{y,k})$ become $\left(e^{i\frac{\pi}{2}}, e^{i\pi}\right)$, $z_{m,k}$ in the equation(6) can be shown in the equation(7) using a thegeometric series of the sensor unit number M, and m is 0.

$$|z_{0,k}| = \begin{cases} 0 & (k = Mn + 1) \\ 1 & (k \neq Mn + 1) \end{cases} \tag{7}$$
$$n \text{ is Positive integer}$$

Figure 3 shows the result of DFT (discrete Fourier transform) analysis to the generated shaft runout and the shaft runout detected by SelfA⁺. As shown in Figure 3, SelfA⁺ does not detect the fourier components of $k = Mn + 1$ in the shaft runout. This result corresponds to the result of the equation (7).

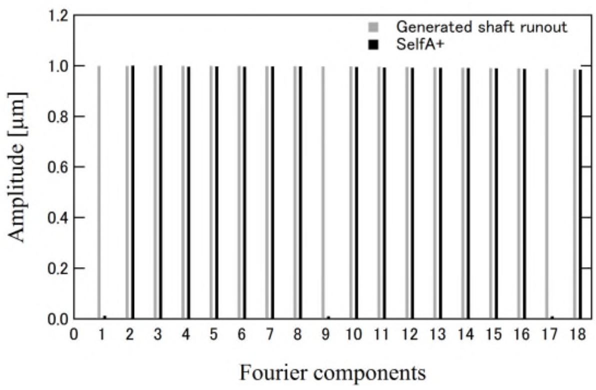

Figure 3. The result of the experiment

5. Conclusion

In this paper, the mathematical principle of SelfA⁺ is quantitatively derived from SelfA, and the shaft runout value evaluated from SelfA⁺ experiment is compared with the value of the mathematical model.

The result shows that the mathematical principle of SelfA⁺ coincides with the value of the experiment. In addition, the similar results is obtained in other generation conditions of the shaft runout which changes the value of the amplitude or the phase. Therefore, the mathematical model is derived and confirmed it.

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