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Considering Aliasing of Resonances in the Design of discrete-time controlled Mechatronic Systems

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Abstract
In high performance mechatronic systems, such as stages or manipulated optics, motion control often suffers from the presence of weakly damped flexible resonances of the controlled mechanisms. These resonances can cause severe stability problems leading to significant performance limitations of the control loop. Since motion control systems are typically implemented in discrete-time, a further phenomenon can hamper the performance - aliasing of resonances. A resonance of the continuous-time plant above the Nyquist frequency appears as a resonance mirrored at the Nyquist frequency in the sampled discrete-time plant seen by the controller. This can equally endanger stability and performance of the control loop. This article gives a description of the phenomenon. It gives guidance on the modelling and mitigation of potentially aliasing resonances of the mechanism. Finally, it discusses the trade-off in the design of anti-aliasing filters – rejection vs. phase lag and shows approaches to reduce the phase lag.

Keywords: Aliasing of resonances, motion control, discrete-time control, weakly damped flexible modes, anti-aliasing filter

1 Introduction

In sampled-data control systems, the issue of aliasing is related to the dynamics of the plant. The design of the plant and of anti-aliasing filters should be targeting at sufficiently rejecting the aliased dynamics in order to maintain the closed-loop stability and performance. Unlike aliasing of signals in sampled-data signal processing, this is rarely discussed in literature [1,2,3].

This article is organised in three parts: the introduction to the phenomenon, guidance on modelling and designing of the mechanism and finally discussion of phase-lag of anti-aliasing filters and of means of its reduction.

2 The phenomenon of aliasing of resonances

The cause of aliasing of dynamics is actually twofold. Firstly, the digital-to-analogue conversion (DAC) at the input of the plant modulates the discrete-time input signal and generates additional signal content above the Nyquist frequency. By sampling at the output of the plant these signals alias back to the original frequency. In this way, the dynamics of the plant above the Nyquist frequency can interact with the discrete-time controller (cp. [2] Fig 1, [1] Fig. 9.3 and Fig. 9.5).

Fig. 1 illustrates the phenomenon. Four different weakly damped (0.1%) continuous-time second-order systems (dashed) are shown - one exhibiting its resonance below the Nyquist frequency $\omega_s/2 = \pi/T_s$ (blue) and three above, whereof two in the band from $\omega_s/2$ to ω_s (green and magenta) and one in the band from ω_s to $3\omega_s/2$ (cyan). The three sampled systems (solid) related to a continuous-time resonance above $\omega_s/2$ show their resonance at the aliased frequency $\omega_0^* \approx \omega_s - \omega_0$ or $\omega_0^* \approx \omega_0 - \omega_s$. The static gain remains unchanged but the damping increases with decreasing aliased resonance frequency ω_0^* . The phase response of all sampled systems shows an additional delay of $T_s/2$ from the zero-order hold (ZOH) of the DAC [1].

The aliasing of the resonances can be explained by poles of the sampled system [2,3]. Systems with continuous-time resonances above the Nyquist frequency $\omega_s/2$ show discrete-time poles at

$$z_{1,2}^* = e^{(-\xi\omega_0 \pm j\sqrt{1-\xi^2}(\omega_s-\omega_0))T_s} \text{ for } \omega_0 \in \left[\frac{\omega_s}{2}; \omega_s\right] \text{ or}$$
$$z_{1,2}^* = e^{(-\xi\omega_0 \pm j\sqrt{1-\xi^2}(\omega_0-\omega_s))T_s} \text{ for } \omega_0 \in \left[\omega_s; \frac{3}{2}\omega_s\right].$$

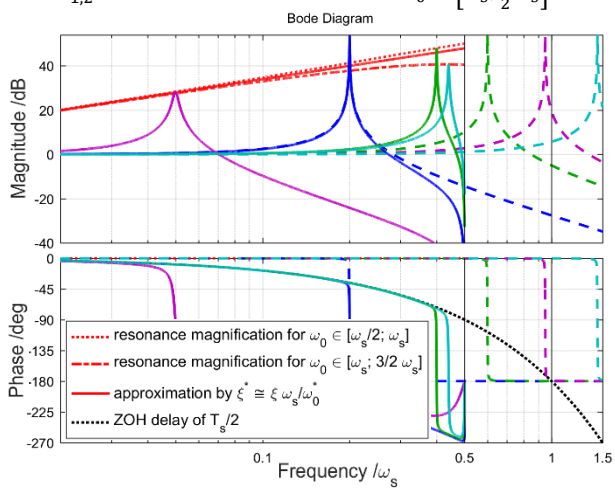

Figure 1: Aliasing of resonances for four different second order systems (dashed: continuous-time, solid: discrete-time, red: characteristics of damping of aliased resonances)

Comparing this to the poles $z_{1,2} = e^{(-\xi\omega_0 \pm j\sqrt{1-\xi^2}\omega_0)T_s}$ of a sampled system originating from a continuous-time resonance below $\omega_s/2$ reveals that the imaginary parts of the continuous-time poles $\pm\sqrt{1-\xi^2}\omega_0$ aliase to $\pm\sqrt{1-\xi^2}(\omega_s-\omega_0)$ respectively $\pm\sqrt{1-\xi^2}(\omega_0-\omega_s)$ whereas the real parts $-\xi\omega_0$ remain unchanged. Thus, for weak damping the equivalent continuous-time resonance is found at $\omega_0^* \approx \omega_s - \omega_0$ respectively $\omega_0^* \approx \omega_0 - \omega_s$ with an increased damping

$$\xi^* \approx \xi\frac{\omega_0}{\omega_0^*} = \xi\frac{\omega_s-\omega_0^*}{\omega_0^*} \text{ or } \xi^* \approx \xi\frac{\omega_0}{\omega_0^*} = \xi\frac{\omega_s+\omega_0^*}{\omega_0^*}.$$

For $\omega_0^* \ll \omega_s$ the damping converges to

$$\xi^* \approx \xi\frac{\omega_s}{\omega_0^*}.$$

I.e. the damping is approximately reciprocally proportional to the aliased natural frequency ω_0^*.

3 Nature, Modelling and Mitigation of potentially aliasing Resonances

Flexible modes of the mechanism of a mechatronic system are known to limit the performance of the control loop. These are typically the global fundamental bending and torsion modes. In contrast, potentially aliasing modes are typically local modes. If they are located at the sensor or actuator related parts of the mechanism, they can show a high controllability or observability leading to high peaks in the continuous-time open loop FRF of the plant. An example depicted in Fig. 2 is a flexibly suspended target of a position sensor.

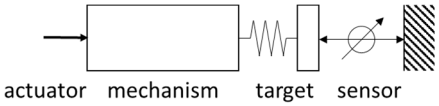

Figure 2: Resonance of a sensor target as an example of a potentially aliasing resonance.

When modelling the controlled mechanism, a FEM model of the mechanism is often imported to Matlab and integrated in the control loop model [4]. Regarding potentially aliasing modes, the imported modes should not be truncated at or below the Nyquist frequency as often done. They should be imported up to ω_s or better $3\omega_s/2$. To reduce the model order, the relevant modes can be selected based on the input-output contribution by determining the Hankel Singular Values from the diagonal dominant controllability and observability gramminans of the modal state space representation [4,5].

The insight in the nature and magnitude of the potentially aliasing modes should be used to improve the mechanical design of the mechanism in order to mitigate the effect of potential aliasing.

4 Anti-Aliasing Filter Design

Anti-aliasing filtering is inevitable to reject aliasing of resonances. However, its phase lag deteriorates the control loop performance.

4.1 Equivalent delay and budgeting of phase lag

The key to quantifying the impact of anti-aliasing filtering on the control loop performance is the concept of *equivalent delay*. It approximates the phase lag of a filter by a single delay number (compare Fig. 3). At frequencies ω well below its poles and zeros any continuous-time filter $F(j\omega)$ shows an almost linear phase $arg\big(F(j\omega)\big) \approx T_e\omega$. The equivalent delay simply is

$$T_e = \sum_{i=1}^{N_z} \frac{1}{Re(s_{zi})} - \sum_{i=1}^{N_p} \frac{1}{Re(s_{pi})},$$

where s_{pi} are the N_p poles and s_{zi} are the N_z zeros of $F(s)$.

Table 1 Equivalent delay budget of a fictitious discrete-time control loop at 5 kHz sampling rate

	Approximation	***Equivalent delay***
Computation time	*One sample step T_s*	*200 µs*
ZOH	*$T_s/2$ [1]*	*100 µs*
Actuator's current control loop	*Complementary sensitivity function*	*40 µs*
Sensor filter	*1st order low pass at 5 kHz*	*32 µs*
Anti-aliasing filter	*2nd order Butterworth at 2kHz*	*113 µs*
Total		***485 µs***

The concept of equivalent delay further allows quantifying, comparing and budgeting all sources of phase lag in the discrete-time control loop. The effect of all sources on control loop can be modelled by the total equivalent delay. Tab. 1 gives an example.

4.1 Selecting the filter order

When designing an anti-aliasing filter the choice of the filter order is usually not obvious. It depends on the required amount of rejection at a critical frequency. Fig. 3 compares Butterworth filters of different order. To equalise the equivalent delay of the filters the cut-off frequency is raised with increasing order. The required factor is given in Fig. 3. The frequency response functions (FRFs) show that from a certain required rejection on it becomes advantageous to increase the order of the filter.

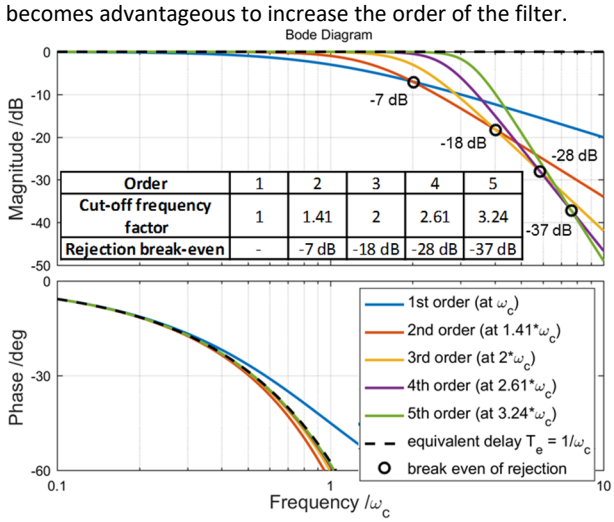

Figure 3: Butterworth filters of different order with equal equivalent delay

4.2 Reducing the total equivalent delay

The understanding of equivalent delay opens several options to further reduce the phase lag of the discrete-time control loop after the anti-aliasing filter is designed to the required rejection.

Firstly, reducing the damping of the poles of the filter reduces the equivalent delay, since the equivalent delay depends on the real parts of the poles. However, the control loop has to be able to deal with the increased resonance magnification of the filter.

Secondly, a notch filter on the critical aliasing resonance can be used instead of or additionally to a relaxed low-pass filter*. Since the poles of a notch-filter are at a much higher frequency than those of a low pass filter the equivalent delay is reduced.

Thirdly, the anti-aliasing filter can be absorbed in the sensor and actuator filters and control loops. Usually, these filters are designed for relatively high bandwidth in order to reduce the impact on the control loop. However, they show equivalent delay but do not contribute effectively to anti-aliasing rejection (cp. Tab. 1). If the filters respectively control loops of sensors and actuators are designed such that they implement the desired anti-aliasing filter characteristic directly their initial equivalent delay contribution is omitted.

Finally, discrete-time controllers for motion control typically include discrete-time low pass filters. This low pass filter does not contribute to anti-aliasing rejection. If this filter or a part of it (the remaining discrete-time controller must stay proper) is shifted to the continuous-time section of the control loop, the open loop FRF remains unchanged while the anti-aliasing rejection is increased.

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