

## A comparative study of curvature-based registration methods for dimensional metrology

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### Abstract

Registration is one of the most important and decisive steps in quality assessment for dimensional metrology. It is basically used to transform the measurement data into a common coordinate system and obtain the complete measured shape. This operation is achieved by means of two steps, namely coarse and fine registration. In this paper, discrete curvature-based registration methods are developed to enhance coarse and fine registration. For fine registration, three variants of Iterative Closest Points (ICP) method are developed and the error metrics are compared. Performances of these methods are evaluated on simulated data and the obtained results show the effectiveness of these methods.

Dimensional Metrology, Registration, Iterative Closest Points (ICP), Point-to-Point (P-P) Minimization, Point-to-Plane (P-Pl) Minimization, Lp-Norm

### 1. Introduction

In precision metrology, registration is a crucial issue especially for the measurement of freeform shapes. It aims at transforming the respective data into a common coordinate system and then rebuilding the complete shape. This allows to recover information from measured data. The registration problem could be formulated as follows: find the optimal transformation  $(R, T)$  ( $R$  the rotation matrix,  $T$  the translation vector) from the common parts or overlaps of data called correspondences. This operation is achieved in two steps: First, a coarse registration is applied on data in order to get an initial rough alignment. Second, a fine registration is then applied to refine the alignment.

Coarse registration could be achieved using many techniques: automated shape alignment [1], marker-based approach [2], genetic algorithms methods [3], principal component analysis (PCA) [3], etc. The last method has been robustified recently using the least-median of squares method (LMedS) for the principal axis determination with the presence of outliers but no standard method has been adopted for coarse registration [4].

Fine registration is performed using very popular methods based on Iterative Closest Point (ICP) algorithm or its different variants [5]. The ICP algorithm can be divided into four main stages: closest point matching, rigid transformation estimation, error metrics determination and stop criteria.

This work presents an automated registration method using discrete curvatures. Coarse registration is made using the Hough Transformation (HT). This method has the advantage of performing registration without the need of established known correspondences between measured and model data. It requires neither markers nor user interaction. In fact, it exploits the natural existing markers on the surface which are described by curvature parameters. For fine registration, two variants of ICP algorithm were explored in addition to the classical point-to-point version: point to plane and a combined version of the

two. In those versions, finding correspondence is also based on surface parameters. The proposed methods were applied on simulated data to demonstrate their effectiveness.

### 2. Approach

The curvature parameters are calculated first for the considered data sets (measured and model data). Principal curvatures are obtained using a tensor-based method [6]. Other parameters needed for the registration process are deduced from curvature values. Those parameters allow also to define the surface type at each point from a set of 10 predefined surface types [6].

#### 2.1. Coarse registration

The proposed coarse registration approach is based on the classical Hough transformation (HT) [7] where local transformation is calculated between each point in model data ( $Q$ ) and each point in measured data ( $P$ ). Those transformations are classified in the Hough table and the one with the highest counter value is selected. One major drawback of this method is the complexity. In fact, it is estimated to be  $O(N \times M)$ , where  $N$  and  $M$  are the numbers of points in  $P$  and  $Q$  respectively. Our method enables to calculate transform only between points with the same surface class according to the pre-calculated values of curvature parameters. In this way, the complexity could be reduced to:  $O(\sum_{i=1}^10 N'_i \times M'_i)$ , with  $N'_i$  and  $M'_i$  denoting the numbers of points of the same surface type  $i$  in  $P$  and  $Q$  respectively.

#### 2.2. Fine registration

The general scheme of an ICP algorithm is composed of two main steps: first, finding corresponding points in the overlapping area and then an optimization process is implemented in order to determine the transform. The first step affects roughly the quality of the result. Correspondence is searched among points with the same curvature and the same surface type using a criterion based on geometric distances. For

the optimization process, the three variants are: Point-to-point (P-P) (eq.1), point-to-plane (P-PI) (eq.2) and a combined version (C) of the previous two (eq.3). Figure 1 illustrates the steps of registration process.

$$\min_{R,T} f(R,T) = \frac{1}{n} \sum_{i=1}^n \|Rp_i + T - q_i\|^2 \quad (1)$$

$$\min_{R,T} f(R,T) = \frac{1}{n} \sum_{i=1}^n \left( (Rp_i + T - q_i)^T n_i \right)^2 \quad (2)$$

$$\min_{R,T} f(R,T) = \sum_{i=1}^n \frac{1}{n} \left[ \alpha_i \|Rp_i + T - q_i\|^2 + \beta_i \left( (Rp_i + T - q_i)^T n_i \right)^2 \right] \quad (3)$$

In the above equations, R, T denote the transformation parameters,  $p_i$  and  $q_i$  the corresponding points in P and Q respectively, n the number of data points and  $n_i$  the normal vector to the tangent plane at  $q_i$ . In eq.3,  $\alpha_i$  and  $\beta_i$  are two weighting parameters set according to the calculated surface type value for each point.

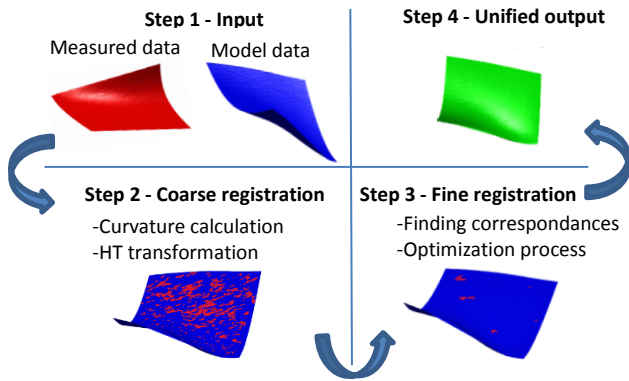


Figure 1. Steps of registration process

### 3. Results

To demonstrate the benefits of our method, it was applied on simulated data sets with different levels of noise in order to simulate different measuring conditions and estimate the robustness of the algorithms. The proposed algorithms were implemented on a personal computer based on Intel Core i7/x64 platform with 8 GB of RAM and a 2.40 GHz processor.

The simulated data were obtained by combining a Gaussian noise with controlled mean and standard deviation to industrial complex shapes (LNE artefact and a LEGO® connector). Three simulations with the following noise parameters were generated ( $\mu=0$ ,  $\sigma=5$ , 50 and 100 nm) and used for the three case studies (fig.2). Those parameters correspond to the typical noise observed in different measuring coordinates systems. The results are evaluated according to a criterion of  $L_p$  norm.

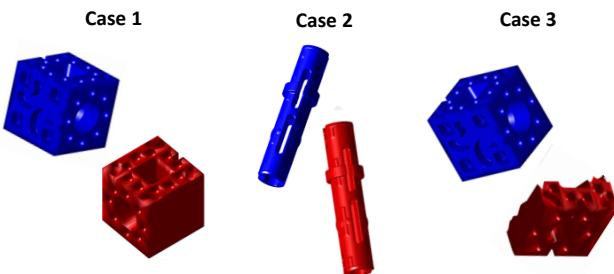


Figure 2. Industrial case studies. (Blue: Model data, Red: Measured data). Case 1: LNE artefact (measured, model: 18053 vertices). Case2: LEGO® connector (measured, model: 53931 vertices). Case3 (measured: 7996 vertices, model: 18053 vertices)

For the registration results, cases 1 and 2 lead to a perfect alignment of the two data sets. However, for case 3, which is partially overlapping, the results are less accurate but still satisfactory.

In order to find the optimal  $p$  that gives the lowest error value, calculations were performed on 2 cases out of the three using  $L_1$ ,  $L_2$  and  $L_\infty$  norms according to the second variant of the algorithm with a superposed noise with  $\mu=0$  and  $\sigma=100$ nm. Results show that  $L_\infty$  gives the lowest value (table 1). Nonetheless, the number of matched points for  $L_2$  is the highest especially for partially overlapping cases. For this reason, it was concluded that  $L_2$  is the suitable method that leads to high quality registration.

In a previous study [6], results show that the point to plane (P-PI) variant outperforms point ot point (P-P). The combined method (C) gives better results than (P-PI) but this superiority is not justifiable taking into consideration its calculation time.

Table 1 Fine registration result using point to plane (P-PI) method with added noise:  $\mu=0$  and  $\sigma=100$ nm to measured data

	Case 1		Case 2	
	Error	Nbr of Corre. points	Error	Nbr of Corre. points
$L_1$	3.05e-6	18053	1.79e-6	53931
$L_2$	2.57e-8	18053	1.17e-8	53931
$L_\infty$	1.22e-9	18053	2.06e-9	53931

### 4. Conclusion

The paper presents an automated curvature-based method for registration of two data sets in dimensional metrology. An approach based on the Hough Transformation was exploited to perform coarse registration. For fine registration, three ICP variants based on curvature parameters were proposed.

The three variants were tested on data with different noise levels under various measuring conditions. Among the three variants, the second one seems to be the most convenient regarding its registration accuracy and computation time. It was shown that the optimum value of  $p$  for  $L_p$  is 2. In fact,  $L_2$  ensures the maximum matching points.

Future work will consider further comparative study and validation tests on optical and CT measurement sets of freeform shapes. Moreover, reference and measured data sets will be used in order to verify and validate the proposed method. Improvement of ICP variant by exploring method such as Kd-Tree in order to reduce computational time will be also investigated.

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