

## A mathematical explanation for the grinding marks in cross and parallel grinding

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### Abstract

Cross and parallel grinding are two common grinding modes for the ultra-precision grinding of axis-symmetrical surface. However, grinding marks often remain on the workpiece machined by cross or parallel grinding, which would deteriorate the surface accuracy and optical performance. In this study, the influence of grinding parameters on grinding marks was analysed and a mathematical formula for the distribution of grinding points was proposed to simulate the grinding marks resulting from cross or parallel grinding. Finally, in order to optimize the grinding marks, a common method for selecting the optimal grinding parameters was proposed. This optimizing method was successfully applied in the ultra-precision grinding of Fresnel micro-structured surface on tungsten carbide, which not only weakened the grinding marks, but also improved the surface roughness.

Key words: grinding marks; mathematical explanation; cross grinding; parallel grinding; Fresnel microstructure

### 1. Introduction

Cross and parallel grinding are two common grinding modes for the ultra-precision grinding of axis-symmetrical surface, like aspherical lens and Fresnel lens. However, grinding marks often remain on the ground workpiece, which would deteriorate the surface accuracy and optical performance. In this study, the relationship between grinding marks and grinding process parameters was analysed. Besides, a common mathematical formula was developed for optimal grinding parameters matching and optimized grinding marks. Finally, this optimizing method was utilized in Fresnel microstructure grinding process successfully, which not only weakened the grinding marks but also improved the surface roughness.

### 2. Relationship between grinding parameters and grinding marks patterns

Huo et al. [1] indicated that it was the run-out error of grinding wheel that caused non-uniform material removal circumferentially and led grinding marks in silicon wafer grinding. Similarly, the origins of grinding marks in cross grinding and parallel grinding are also grinding wheel' run-out error. As for the grinding wheel used in this study (Fig 1), the measurement result by a laser displacement sensor showed that it existed an run-out error of about  $3\mu\text{m}$ . The run-out error could never be thoroughly removed because of the non-uniform wheel wear during the grinding process, thus the grinding marks couldn't be eliminated completely. However, it could be optimized by selecting proper grinding parameters.

Figure 2(a) and 3(a) illustrated the influence of wheel run-out error and grinding parameters selection on grinding marks. The material removal area during one revolution of grinding wheel would be non-uniform circumferentially, remaining a circumferential surface with different height (red means high area and blue means low area). Besides, different grinding parameters selection would result in different surface topography. In Fig 2(b), because of improper parameters matching, the high areas on each revolution of the workpiece

would coincide at the same angular position, which would generate remarkable grinding marks. However, in Fig 3(b), the high area of the first revolution would meet the low area of the next revolution at the same angular position, then the residual high area of the first revolution may be removed during the second revolution of the workpiece, thus the resultant grinding marks would be weakened.

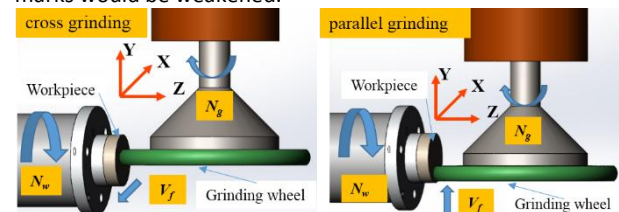


Figure 1. Kinematics illustration of parallel grinding and cross grinding

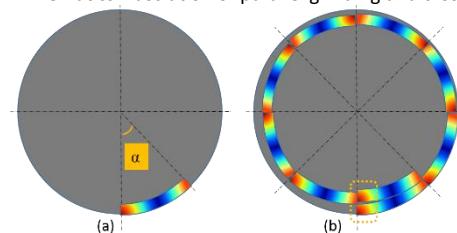


Figure 2. non-uniform material removal in the circumferential direction during parallel grinding- improper grinding parameters

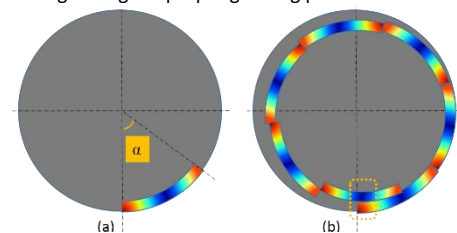


Figure 3. non-uniform material removal in the circumferential direction during parallel grinding- proper grinding parameters

In order to show the grinding marks features more clearly, the ground area during wheel's one revolution would be represented by one grinding point. During the parallel grinding process of a spherical surface, for each grinding point  $P_n(X_n, Y_n, Z_n)$ , its three-dimensional coordinate could be calculated by the formula (1-1). Besides, this formula is also

suitable for cross grinding. In Fig 4, a grinding marks pattern was calculated using this formula, which accord perfectly with the actual grinding marks ( $N_w=120, N_g=5999, R=500, V_f=5$ ).

$$X_n = R \cdot \sin\left(\frac{(N-n) \cdot v_f}{N_g \cdot R}\right) \cdot \cos\left(\frac{2\pi \cdot n \cdot N_w}{N_g}\right) \quad (1-1)$$

$$Y_n = R \cdot \sin\left(\frac{(N-n) \cdot v_f}{N_g \cdot R}\right) \cdot \sin\left(\frac{2\pi \cdot n \cdot N_w}{N_g}\right)$$

$$Z_n = R \left(1 - \cos\left(\frac{(N-n) \cdot v_f}{N_g \cdot R}\right)\right)$$

$n$ : the order number of grinding point Pn

$R$ : radius of the spherical surface (mm)

$N_g$ : grinding wheel rotation speed (rpm)

$N_w$ : workpiece rotation speed (rpm)

$V_f$ : federate of grinding wheel (mm/min)

$N$ : the sum of grinding point generating on the workpiece

$$N = \frac{l \cdot N_g}{v_f} \quad (1-2)$$

$l$ : the feeding length (mm)

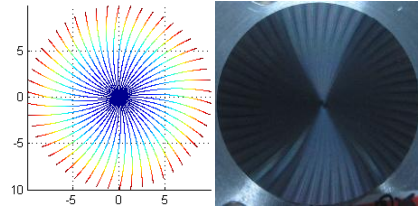


Figure 4. parallel grinding of silicon carbide plane surface (emulational grinding marks and actual grinding marks)

### 3. The method for optimizing grinding marks

As shown in fig 5, according to the study of Chen[2], to optimize the grinding marks, we need to facilitate  $d_r \approx d_c$ . This could be achieved by choosing proper grinding parameters depending on formula (1-3~1-5).

$$d_c = \frac{2\pi r}{t} \quad (1-3) \quad d_r = \frac{v_f \cdot t}{N_g} \quad (1-4)$$

$$\text{if } d_r = \frac{v_f \cdot t}{N_g} = \frac{2\pi r}{t} = d_c \quad (1-5)$$

$$\text{we get } t_{\max} = \sqrt{\frac{2\pi r \cdot N_g}{v_f}}$$

$t$ : the number of ray in grinding marks

$r$ : radius of the workpiece (mm)

Besides,  $t$  is also determined by the ratio  $r_{gw}$  of grinding wheel speed and workpiece speed. For example, according to Fig 4 and 5, we could found that  $t=51$  when ( $N_w=120, N_g=5999, R=500, V_f=5$ ) and  $t=151$  when ( $N_w=120, N_g=6040, R=500, V_f=5$ ). There is an approximate relationship between this numbers:

$$t=50 \approx 5999/120 = 1 \cdot r_{gw}; \quad t=151 = 3 \cdot (50 + 1/3) = 3 \cdot (6040/120) = 3 \cdot r_{gw};$$

In fact, this approximate relationship could be proved mathematically but I don't have enough space in this paper. Anyway, using the formula (1-3~1-5) and the relationship between  $t$  and  $r_{gw}$ . The optimal grinding parameters were obtained as ( $N_w=120, N_g=6048, R=500, V_f=5$ ), as shown in Fig 6.

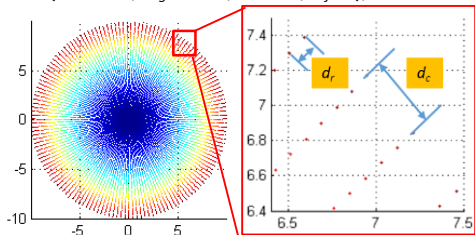


Figure 5. emulational grinding marks ( $N_w=120, N_g=6040, R=500, V_f=5$ )

### 4. Optimizing the grinding marks of Fresnel microstructure grinding

This optimizing method for grinding marks could also be used in the grinding process of complex axis-symmetrical surface. Fig 7 and 8 showed the successful application of this optimizing method in ultra-precision grinding of Fresnel micro-structured surface, which not only weakened the grinding marks, but also improved the surface roughness, from Ra 93.9nm to Ra 6.4nm.

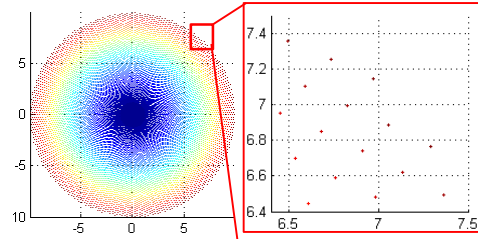


Figure 6. optimized grinding marks ( $N_w=120, N_g=6048, R=500, V_f=5$ )

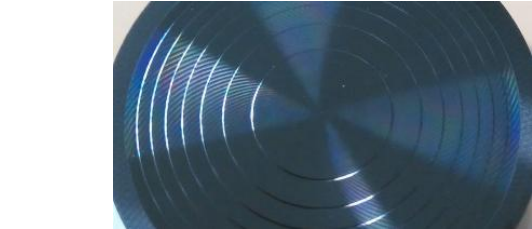
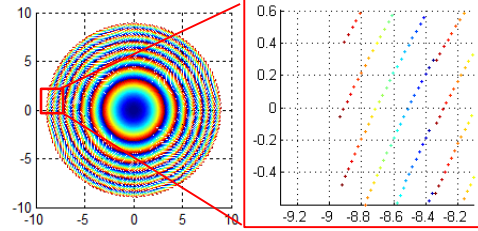


Figure 7. Fresnel microstructure by parallel grinding-with bad grinding marks-WC-Ra 93.9nm--(6045rpm-500rpm-1mm/min-R100mm-r10mm)

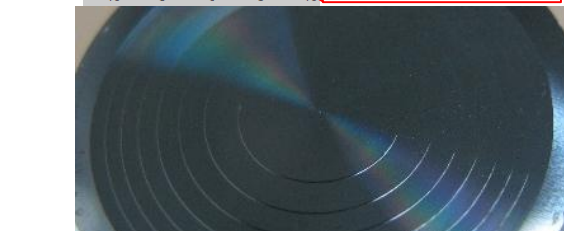
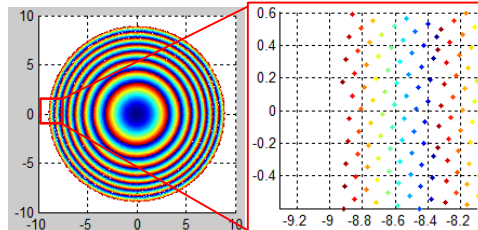


Figure 8. Fresnel microstructure by parallel grinding-with bad grinding marks-WC-Ra 6.4nm--(6045rpm-498rpm-1mm/min-R100mm-r10mm)

### 5. Conclusion

Through the developed formulas to simulate the grinding marks and the method for optimizing grinding marks, this paper have given a mathematical explanation for the grinding marks in cross and parallel grinding.

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### References

- [1] Huo F W, Kang R K, et al. 2013 INT J MACH TOOL MANU 66 54-65
- [2] Chen B, Guo B, et al. 2015 EUSPEN conference 79-80