Magnification Dependent $MPE$-equation for Dimensional X-ray CT Metrology

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Abstract

It is customary among manufacturers of X-ray computed tomography (CT) equipment for dimensional metrology to claim the accuracy (‘trueness’ and precision) of their measuring systems with statements of maximum permissible error of the form $MPE = (A + L/K) \mu m$, with $A$ and $K$ being constant factors and $L$ a dimension length (in mm) associated with the size of the measuring workpiece. This paper shows that these type of $MPE$ statements might be only adequate for uni-directional (centre-to-centre) distances, effectively predicting small deviations in the CT measurements from typical reference (‘operative true values’) data obtained from coordinate measurement machines (CMM), and in such a case with deviations approximately independent of the magnification used during the CT scans. However, these $MPE$ statements do not directly account for the increase of dimensional errors for measurements of diameter, roundness, or bi-directional lengths with CT when, indirectly, larger specimens require smaller magnifications if optimal parameter settings are used for image acquisition. In that case, it should be necessary to add a magnification dependent factor to the $MPE$ statement. Adding a term of the form $X/B$, with $X$ as the magnification axis position of the CT scanner and $B$ another constant factor (determined for a particular CT setup and calibration reference object), could account for larger measurement errors when smaller magnifications are used for CT scans. Thus, the total maximum permissible error for measurement of length with CT should be of the form $MPE_{X} = (A + X/B + L/K) \mu m$. This is demonstrated through the analysis of experimental results from a study that investigated dimensional measurements deviations between CT and CMM data when the magnification for CT data collection changes. A cone-beam CT setup was used for the study, and measurements were taken on two metallic hole-plates provided by the National Metrology Institute of Germany (PTB). The dimensioning of the hole-plates shows that the CT uni-directional measurements converge to measured values with uncertainties (computed with the substitution method from the VDI/VDE 2630-2.1 guidelines) within the 3 to 5 \( \mu m \) range, while the CT bi-directional measurements yield measured values with larger uncertainties.

Keywords: Dimensional metrology, Computed tomography (CT), Magnification, Uncertainty, Accuracy, Precision, Maximum permissible error (MPE).

1. Introduction

In a typical industrial cone-beam X-ray CT setup (e.g., see Figure 1), a two-dimensional image is created by the projection of the X-rays on a flat panel detector when X-rays emanating from a diverging cone-beam source transmit through a three-dimensional object. Considering an ideal point source, and based upon similar triangles, the ratio of the image size $y_2$ to the object size $y_1$ for a cone-beam CT system can be computed as

$$M = \frac{y_2}{y_1} = \frac{SDD}{SOD}, \quad (1)$$

with SDD as the source-to-detector distance and SOD as the source-to-object distance. The ratio from Eq. (1), is usually called the ‘geometric magnification’. This paper presents preliminary analysis of experimental results from a study that investigated X-ray CT dimensional measurement deviations, from CMM reference data, when the magnification $M$ for CT scanning changes with variations of SOD but SDD is fixed (unchanged). Two workpieces provided by PTB were used for this study, a hole-plate made from aluminium (nominal size 48 mm x 48 mm x 8 mm) and a smaller hole-plate made of steel (nominal size 6 mm x 6 mm x 1 mm), see references [2, 3] for details; and highly accurate measurements (with uncertainties less-than or approximately equal to 1 \( \mu m \)) obtained by tactile CMM were used as references to define the ‘operative true values’ associated to measurands of interest in the hole-plates.

![Figure 1. Basic geometry of experimental setup for CT data collection.](image)

2. CT data collection

Figure 1 shows the geometry of the experimental setup used. Each hole-plate has 28 drilled holes, and the measurands of interest correspond to distances between the holes at the mid-plane (i.e., on a plane located between the top and bottom surfaces) of the plates—using as reference features the circles of intersection between such a plane and the holes [2]. A total of 165 characteristics were measured on each hole-plate, 62 uni-

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1 The compatibility/incompatibility of ‘error-based’ and ‘uncertainty-based’ modeling of measurement is still being debated [1] and is beyond the scope of this paper due to the limited space.
directional (centre-to-centre) lengths, 47 bi-directional (point-to-point) distances, and the diameters and respective roundness values of the 28 reference circles at the mid-plane. CMM reference values for the measurands of interest were obtained with a Zeiss Micura CMM system, while CT measurements were taken with a Zeiss Metrotom 800 X-ray CT machine. The CT scans were obtained at different positions of the magnification X-axis for each sample, see Figure 1. These positions were linked to the SOD parameter by small offset, \( \text{SOD}(mm) = X + 15.101 \text{ mm} \), while on the other hand SDD was kept fixed at 805.21 mm.

3. Experimental Results

![Figure 2](image.png)

Figure 2. Mean deviations of dimensional measurements on the PTB hole-plates. Vertical bars cover two standard deviations from the mean.

Figure 2 shows some preliminary results. Data for \( X \) positions from 25 to 175 mm correspond to the steel small plate, and data between the 200 to 500 mm correspond to the large aluminum plate. For the steel plate item, the smallest deviations between CT and CMM data for the overall set of measurement categories (uni-directional/bi-directional lengths, circles diameters and roundnesses) were obtained with CT scans performed at the largest magnification, \( M = 16.9 \), or smallest voxel size \( V_X = 8 \mu m \), which corresponds to the nominal magnification axis position \( X = 25 \text{ mm} \). With a decrease in magnification, or equivalently an increase of the voxel size for the CT scan, the deviations between CT and CMM data increase for all the categories of measurements tested except the uni-directional lengths. On average, the center-to-center distances remained unaffected by variations of CT magnification, exhibiting a mean deviation from CMM of \( \Delta = 0.7 \mu m \) (with a standard deviation of 0.8 \mu m) over all the magnifications tested and obtained from 434 measurements. This includes 62 uni-directional lengths, each one evaluated at the seven magnification positions listed in Figure 2. Similarly, for the aluminum plate item, the smallest deviations between CT and CMM data for the overall set of categories was obtained at the largest CT magnification, \( M = 3.7 \), or smallest voxel size \( V_X = 33.9 \mu m \), which corresponds to the axis position \( X = 200 \text{ mm} \). A decrease in CT magnification made deviations between CT and CMM for the measurements of roundness larger, but the uni-directional length, bi-directional length, and diameter measurements remained unchanged on average. However, the dispersion of CT measurements clearly increased except for the uni-directional lengths. On average, the center-to-center distances in the aluminum hole-plate exhibited a mean deviation from CMM of \( \Delta = 1.2 \mu m \) (with a standard deviation of 2.2 \mu m) from a total of 434 measurement results. Lastly, a full analysis of uncertainties for all the measurements presented above was also performed, but it will be reserved for a future publication given the limitations of space here.

4. Magnification dependent MPE-relation

From plotting \((\bar{X}_{CT} - \bar{X}_{CMM}) \) vs \( X \), Figure 2 suggests that dimensional X-ray CT measurements experience strong dependency on the magnification parameters used for data acquisition. In CT, the magnification parameter \( M \) is related to the imaging voxel size \( V_X \) through an inverse relationship \([V_X = P_D / M, \text{where } P_D \text{ is the detector effective pixel size}]\). At the same time, the voxel size is a determinant factor that influences the so-called ‘metrological structural resolution’ defined by the smallest feature that can still be measured dimensionally in a 3D CT data set. So, it should not come as surprise that dimensioning a workpiece with X-ray CT would lead to larger uncertainties in the results when larger voxel sizes (or lower magnifications) are used for CT scanning, especially if dimensional measurements are attempted with only two single data-points—the case of bi-directional (point-to-point) measurements above. This happens because dimensional information at a scale smaller than the voxel size is usually lost in the 3D matrix of CT grey values when a single-voxel is used. Despite this phenomenological hunch, in the currently literature, the maximum permissible error (MPE) for dimensional CT measurements is usually quoted with statements of the form \( \text{MPE} = (A + L/K) \), with \( A \) and \( K \) constants and \( L \) the length dimension (e.g., see [4, 5]). This expression contains a length dependent component that expresses the geometrical errors associated with the size of the measuring workpiece, but it does not contain a magnification dependent component. Based on evidence from the experimental data referred in this paper, only the unidirectional (centre-to-centre) distances seem to be independent of the magnification used for CT scan. However, for measurements of diameter and roundness, and bi-directional lengths, the deviations between CT and CMM data are larger and strongly depend on magnification parameters. To account for this, a magnification dependent factor should be added to the MPE statement. The authors of this paper propose to add a term in the form \( X/B \) to cover for all dimensional measurement types, with \( X \) as the magnification axis position of the CT scanner (in mm) and \( B \) another constant factor.

5. Conclusions

In general, decreasing the magnification or, equivalently, increasing the voxel size for X-ray CT scanning a workpiece leads to larger deviations (and uncertainties) in the dimensional data obtained from CT as compared to ‘operative true values’—namely CMM data. This is consistent for several measurements (e.g., bi-directional lengths) when dimensioning the two hole-plates provided by PTB. To account for this, the authors of this paper propose to include a magnification factor so that the total maximum permissible error for dimensional measurements with CT is expressed as \( \text{MPE}_E = (A + X/B + L/K) \) with parameters defined in the text above.

References