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The isostatic 3-body problem: a complete solution

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Abstract

Kinematic couplings are used when two rigid bodies need to be repeatedly and accurately positioned with respect to each other. They allow for sub-micron positioning repeatability by suppressing play and reducing strains in the bodies. Typical applications are are in astronomy, semiconductors and metrology. This work generalizes two-body kinematic couplings to multi-body kinematic mounts with a focus on the 3-body case. This work presents a complete analysis of the 3-body case by providing a novel set of representative examples, realized demonstrators, and applications of 3-body kinematic mounts.

Kinematic coupling, Kinematic mount, Contact point, Exact-constraint, Constraint line, Alignment method, Precision fixturing, Precision engineering.

1. The isostatic 3-body problem

An isostatic assembly is a configuration of bodies in which each degree of freedom between the bodies is constrained exactly once.

Kinematic couplings are isostatic assemblies of two bodies using six contact points and their nesting forces. Kinematic couplings have been extensively researched [1] and applied to wafer scanners [1], AFMs [2] and high precision microscopes [3].

By analogy, and as opposed to the classical unsolved 3-body problem of celestial mechanics, we propose kinematic mounts as a complete solution to the isostatic 3-body problem. Figure 1 shows a kinematic coupling and a kinematic mount. After presenting some examples of kinematic mounts, we continue with results from experiments followed by an application of kinematic mounts. Our results report and extend work done in the first author's Ph.D. thesis [4].

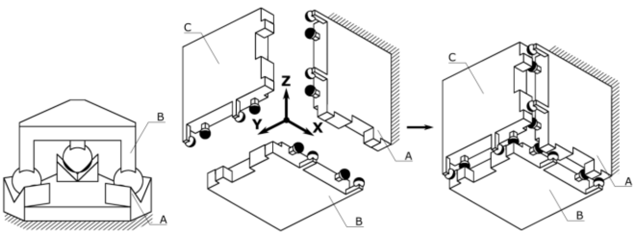


Figure 1. Left: a three-vee kinematic coupling with bodies A and B. Right: three-body kinematic mount configuration C7 with bodies A, B and C in unassembled and assembled state.

2. 3-body kinematic mounts

In order to have a limited list of kinematic mounts, definitions are given to constrain the problem statement, including the concept of equivalent kinematic mounts. This problem statement yields an exhaustive list of seven non-equivalent configurations generated using screw theory as in Whitney [5].

2.1. Definitions

Bodies. Unions of spheres and cuboids whose edges are parallel to the x, y and z axis when assembled.

Constraint line. If P is a contact point of two bodies, then the constraint line is the line containing their normals at P.

Interface. The set I_{AB} of all contact points between two bodies A and B. The interfaces I_{BA} and I_{AB} are considered to be the same.

2.2. Detailed problem statement

To propose examples of kinematic mounts assembled with up to three interfaces whose contact points lie on the axes of a set of orthogonal coordinates (we exclude the intersection point of these axes). All constraint lines are parallel to one of the three orthogonal axes

2.3. Conjectured conditions of 3-body kinematic mounts

There are a total of four conjectured conditions for kinematic mounts:

Condition 1. A 3-body kinematic mount requires 12 contact points.

Condition 2. An interface cannot have collinear constraint lines.

Condition 3. An interface cannot have more than two parallel constraint lines.

Condition 4. Every orthogonal axis has four constraint lines parallel to it.

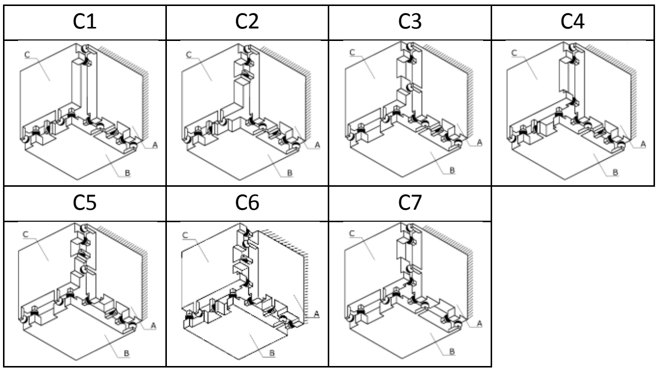
2.4. Equivalence relation for 3-body kinematic mounts

If the orthogonal axes containing the contact points are relabelled, e.g., we interchange the X and Z-axes, we consider the resulting kinematic mounts to be equivalent.

2.5. Examples of 3-body kinematic mounts

The detailed problem statement yields the seven non-equivalent 3-body kinematic mounts shown in Table 1.

Table 1. The seven non-equivalent kinematic mount configurations.



3. Measurements

3.1. Measurements on centimeter-scale metallic mounts

In order to test the performance of kinematic mounts versus kinematic couplings, two prototypes shown in Figure 2, were constructed. The distance between the contact points is of order 1 cm.

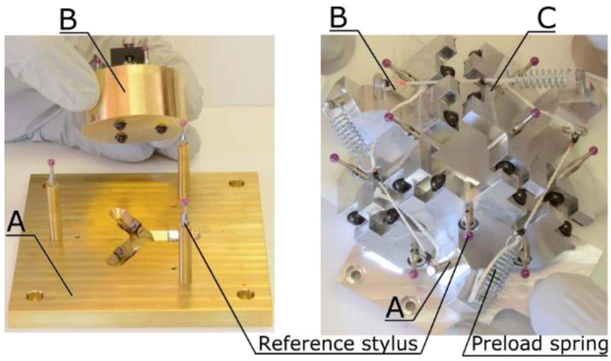


Figure 2. Three-vee kinematic coupling and C7 kinematic mount realized for positioning error measurements.

The contact points are grade three silicon nitride spheres in contact with custom cut steel gage block flats. For our kinematic coupling the preload is provided by gravity, whereas our kinematic mount use preloaded springs. The measurements were done at the Swiss Federal Institute of Metrology (METAS) using the Micro-CMM [6] having a 50 nm measurement uncertainty. For each body three reference styli with ruby spheres were measured to determine their position in the assembled position. The first results of these measurements are given in Table 2. Ten measurements were performed on the three-vee kinematic coupling and three on the 3-body kinematic mount.

Table 2. Average linear and angular positioning error.

	Three-vee	Configuration C7	
Body	B	B	C
Translation error	178 nm	902 nm	712 nm
Angular error	5 µrad	42 µrad	31 µrad

3.2. Second measurements on millimetre-scale silicon mounts.

A second set of measurements (Table 3) was conducted on silicon parts 20x20x0.5mm in size, using a custom developed setup [7]. The preload was applied with small magnets. For configuration C1, C2 and C6 the number of measurements were 27, 23 and 33, respectively.

Table 3. Standard deviation of the linear and angular positioning error.

Configuration	**C1**	**C2**	**C6**
Body	C	C	C
Translation error	4.7 µm	4.1 µm	1.8 µm
Angular error	730 µrad	616 µrad	14 µrad

4. Application example of 3-body kinematic mounts

Kinematic mounts were used to assemble a sugar-cube sized (20 mm edge) delta robot (Figure 3). This improved assembly accuracy as compared to our previous work [8].

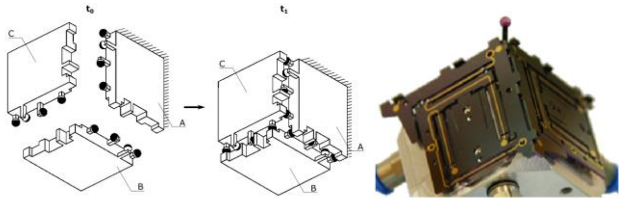


Figure 3. Left: kinematic mount configuration C6 in unassembled and assembled states. Right: sugar-cube size delta robot with kinematic mount configuration C6.

5. Conclusion and future work

This work describes multibody kinematic mounts which generalize well-known two body kinematic couplings. Focusing on the 3-body case, we provide an exhaustive list of seven non-equivalent kinematic mounts, a complete answer to our precise problem statement. This result assumes four conjectures and a notion of equivalence. First experiments show that a kinematic mount has an average positioning accuracy in the micron range. Kinematic mounts were also used to assemble a centimetre-scale silicon delta robot. Kinematic mounts are promising for the future of assembly, however, more experimental evidence is required. The first author's thesis provides detailed information, including a classification of 2-D 3-body kinematic mounts.

6. Acknowledgements

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References

[1] Slocum A H 2010 Kinematic couplings: A review of design principles and applications *Int. J. Mach. Tools Manuf.* **50** no. 4 pp. 310–327

[2] Werner C 2010 *A 3D translation stage for metrological AFM*. Eindhoven (Technische Universiteit Eindhoven)

[3] Hart A J, Slocum A and Sutin J 2004 Segmented and shielded structures for reduction of thermal expansion-induced tilt errors *Precis. Eng.* **28** no. 4 pp. 443–458

[4] Kruis J 2016 Design, analysis, testing and applications of two-body and three-body kinematic mounts Lausanne (EPFL)

[5] Whitney D E 2004 *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development* 1st edition New York (Oxford University Press)

[6] Küng A, Meli F, and Thalmann R 2007 Ultraprecision micro-CMM using a low force 3D touch probe *Meas. Sci. Technol.*, vol. **18**, no. 2, p. 319

[7] Kruis J et al. 2015, *6 DOF repeatability measurement setup for measuring position of assembled silicon parts with nanometric resolution.* Leuven (EUSPEN conference proceedings)

[8] Henein S et al. 2011 Silicon Flexures for the Sugar-Cube Delta Robot Como (EUSPEN conference proceedings)