euspen's 17th International Conference & Exhibition, Hannover, DE, May 2017
www.euspen.eu

A laser based multilateration system for measurement of low-slope surfaces

James Norman, Xavier Tonnellier, Paul Morantz

Precision Engineering Institute, Cranfield University (United Kingdom)

j.p.norman@cranfield.ac.uk

Abstract
Manufacture of metre-scale mirror segments for large telescopes requires measurement of these segments early in the manufacturing process chains whilst the surfaces remain non-specular. A laser based multilateration system is being developed to measure these large low-slope non-specular surfaces. It utilises displacement measurements of a CMM stylus tip using the interferometric mode of laser trackers. To determine the measurement uncertainty of this system a reference artefact has been measured by a high accuracy CMM. The multilateration measurement will be evaluated using this artefact. The experimental evaluation is supported by Monte Carlo simulation using an analytical model to predict the surface coordinate measurement uncertainty of this method. The initial experimental arrangement of the laser based multilateration system measured the z-coordinates of the artefact xy-plane. These have a standard deviation of 13.2 µm, the simulation estimated 11 µm. Simulation of the optimised experimental setup predicts measurement uncertainty σ < 2 µm. Further work will optimise the arrangement of measuring stations and target positions to reduce the measurement uncertainty of this technique.

Keywords: multilateration, non-specular, low-slope, coordinate measurement

1. Introduction

The manufacture of metre-scale mirror segments for large telescopes, such as the European Extremely Large Telescope, requires measurement of these segments early in the manufacturing process chains whilst the surfaces remain non-specular. The uncertainty of this measurement has a direct effect upon the time taken to manufacture these segments; it is therefore critical to the viability of large telescope manufacture that a method for low uncertainty measurement of these surfaces is developed and evaluated. A laser based multilateration system has been proposed to measure these large low-slope non-specular surfaces, it utilises interferometric displacement measurements of a coordinate measuring machine (CMM) stylus tip using laser trackers. We describe the simulation of this measurement system and initial experiments.

2. Laser based multilateration

This section describes the measurement principle of this laser interferometry based multilateration coordinate measurement system; and the methodology for assessing its uncertainty.

2.1. Multilateration

Multilateration is a method for determining a coordinate system about a number of measuring stations and defining a set of measured positions in terms of this coordinate system. Steerable displacement measuring interferometers – the interferometric mode on laser trackers – can be used as measuring stations [1, 2]; whilst a retro-reflecting mirror (retro) is used as a measurement target. By moving the retro between multiple measurement positions, and recording displacement, it is possible to self-calibrate the measurement system to determine the relative positions of stations and targets. This is calculated via multilateration and is described within [3-5]. Measuring at least 10 target positions with at least four displacement measuring stations allows the self-calibration to be calculated using the following equation set:

$$(X_i - x_j)^2 + (Y_i - y_j)^2 + (Z_i - z_j)^2 = (dp_{1j} + d_{ij})^2, \quad (1)$$

where X, Y and Z are the coordinates of the target; i = 1, …, N where N is the total number of target positions; x, y, and z are the coordinates of the measuring stations; and j = 1, …, M where M is the number of stations. The coordinate system has its origin at station one; its x-axis is defined by the vector from station one to station two; the xy-plane is defined by the positions of stations j = 1, 2, 3. dp_{1j} are the interferometric dead paths between each of the measuring stations and the initial target position. d_{ij} are the measured displacements of the target relative to the measuring station positions.

2.2. System arrangement

Takatsuji *et al.* [3] have shown that the z-coordinate of station four must be non-zero in the coordinate system. If it is in the xy-plane, there will be an infinite number of solutions for the set of equations. For a six-station system, a triangular prism station arrangement, with the rectangular face parallel to the measurement positions, produces the lowest uncertainties for the self-calibration; and the measurement points should not approach the xy-plane of the coordinate system. Zhang *et al.* [6] detail a parameter that relates the system arrangement to the self-calibration measurement uncertainty of the system.

2.3. Methodology for determining task specific uncertainty

To determine the task specific measurement uncertainty of this laser based multilateration technique an evaluation in line with the GUM [7] has been undertaken. This follows a Type A experimental analysis and a Type B Monte Carlo simulation.

3. Initial experiment and simulation

This section describes the initial experiment undertaken to begin the Type A analysis of the proposed measurement system. Based on the initial experiment setup, a Type B analysis was carried out.

3.1. Experimental method

A Leica AT960 laser tracker was used as the measuring stations for this experiment; a 12.7 mm aluminium corner cube retro was used as the measurement target. A low expansion glass (ULE®) flat was used as the measurement artefact (0.2 m^2 square measurement surface). The retro was coupled with CMM stylus and installed on a Leitz PMMF-1000 CMM. The CMM was used to probe the surface of the artefact with the retro-stylus enabling direct tracker measurement of the stylus tip's centre. A sequential style of multilateration was utilised: the tracker was moved into six unique measurement station positions whilst the CMM repeated its probing routine on the artefact six times. Figure 1 shows the experimental setup. The stylus probed the surface in a 10 x 10 grid for 100 target positions. The arrangement of the measuring stations and target positions followed the design rules set out in section 2.2. The artefact surface was also measured using the laser tracker in its normal mode and the Leitz CMM for comparison.

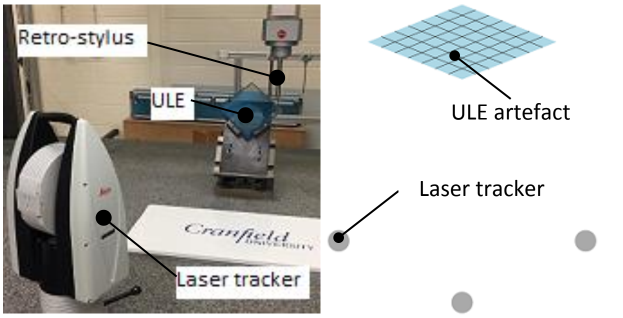

Figure 1. A Leica AT960 laser tracker following a retro-stylus as it probes the surface of a ULE glass flat artefact (left). The stylus is controlled by a Leitz PMM-F 1000 CMM. Top-view of experimental arrangement (right). Each tracker position has another tracker position directly below to form a triangular prism. Artefact tilted at 45°.

3.2. Multilateration measurement simulation

The bespoke Monte Carlo simulation uses a mathematical model of the measurement system implemented in Matlab. By quantifying the uncertainty of the input parameters of, among others: interferometric displacement, CMM positioning, and retro sphericity; and propagating them through a mathematical model of the system to determine an estimate for the output parameters uncertainties, i.e. target coordinate uncertainties and measuring station coordinate uncertainties.

For an idealised experimental setup, strictly following the design rules as detailed within [3, 6], measuring the form of a 1 m^2 low-slope surface, the simulated estimated coordinate measurement uncertainty is $\sigma < 2$ µm ($k = 1$). At this stage, it is not possible for practical experimental reasons, such as a small retro viewing angle (~60º), to achieve the idealised system arrangement – in terms of target and measuring station positions. However, the purpose of this initial experiment was to compare its measurement result with the simulated measurement result of this setup, thus enabling a verification of the simulation.

3.3. Results

Table 1 shows the standard deviation of points, in z, from the nominal xy-plane (σ_z) of the artefact as measured by: the CMM; the laser tracker in normal operating mode; and calculated from the laser-based multilateration. The simulated multilateration measurement result is also listed.

The CMM measurement of σ_z is considered here as the reference measurement; the CMM measurement uncertainty is $\sigma_L \leq 1.3$ µm, where σ_L is the uncertainty of a CMM length measurement ($k = 1$) for this size artefact. The laser tracker measurement in this setup utilised the low uncertainty interferometer effectively due to the orientation of the artefact; hence the relatively small difference between this result and the CMM reference measurement. As expected, the multilateration result for this setup produced a relatively large difference to the result from the CMM; however, the result is less than a factor of two greater than that predicted by the simulation of the multilateration σ_z result. This provides some verification of the simulation and therefore some confidence in the predicted measurement uncertainty achievable with an optimised setup, as discussed in section 3.3.

Table 1. The σ_z of a ULE glass artefact as measured by a Leitz PMM-F CMM, Leica AT960 laser tracker, calculated by multilateration, and Monte Carlo simulated σ_z measurement.

Measurement system	σ_z/µm
CMM	2.3
Laser tracker (k =1)	4.0
Multilateration (k = 1)	13.2
Simulation (optimised setup, k = 1)	< 2
Simulation (initial experiment setup, k = 1)	11

5. Conclusions

A laser based multilateration measurement system has been proposed for the measurement of large non-specular low-slope surfaces. An analytical model of this system has been developed. Initial experiments have tested this technique and provided some verification of the analytical model simulation. The result calculated via multilateration was 13.2 µm. This is in comparison to the simulated result of 11 µm, providing some confidence in the simulation. Simulation of the optimised setup predicts the coordinate uncertainty $\sigma < 2$ µm.

Further work will include: a more detailed explanation of the multilateration system, its design rules, and evaluation of its input error sources and resulting measurement uncertainties; and optimisation of the positions of measuring stations and targets in order to reduce the uncertainty of this measurement technique. This will be enabled by producing a high angle retro in order to increase the measuring area. The optimised result from laser multilateration will be combined with the CMM result to further reduce the artefact measurement uncertainty.

Acknowledgements

We would like to thank the Centre for Doctoral Training in Ultra Precision Engineering and Hexagon Manufacturing Intelligence for funding this research and supporting the authors.

References

[1] Greenleaf A H 1983 Self-calibrating surface measuring machine. *Opt. Eng.* **22**(2) 222276

[2] Hughes E B, Wilson A, and Peggs G N 2000 Design of a High-Accuracy CMM Based on Multi-Lateration Techniques, *CIRP Annals - Manufacturing Technology.* **49**(1) 391-394

[3] Takatsuji T *et al* 1998 Restriction on the arrangement of laser trackers in laser trilateration *Meas. Sci. Technol.* **9** 1357-1359

[4] Camboulives M, Lartigue C, Bourdet P, Salgado J 2016 Calibration of a 3D working space by multilateration *Precision Engineering.* **44** 163-170

[5] Wendt K, Franke M, Härtig F 2012 Measuring large 3D structures using four portable tracking laser interferometers *Measurement*. **45**(10) 2339-2345

[6] Zhang G X *et al* 2003 A Study on the Optimal Design of Laser-based Multi-lateration Systems *CIRP Annals - Manufacturing Technology*. **52**(1) 427-430

[7] BIPM, IEC, ILAC, IFCC and IUPAC and OIML. 2008 Evaluation of measurement data - Guide to the expression of uncertainty in measurement, International Organization for Standardization