

## Integral and traceable evaluation of three-dimensional thread measurements

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### Abstract

Improving the calibration procedure of thread gauges and rings is of particular interest to achieve traceable results for thread parameters with competitive measurement uncertainty.

In order to obtain more information about complex geometries, three-dimensional approaches to measurement as well as its evaluation are necessary. A three-dimensional dataset of a helical geometry gathered by a coordinate measuring machine (CMM) has to be evaluated according to its continuous 3D surface which is described by a parametric representation. The parameter determination of helical geometries leads to a non-linear  $L_2$ -approximation problem. Different mathematical approaches, e.g. the Gauss-Newton method and the Levenberg-Marquardt method, provide an efficient solution to  $L_2$ -approximation problems.

Furthermore, the  $L_2$ -approximation approach allows us to take into account measurement uncertainties of each measuring point for the error propagation according to type A of the Guide to the expression of uncertainty in measurement (GUM). Finally, the algorithm determines the six degrees of freedom and form parameters of complex geometries with their corresponding uncertainties.

coordinate metrology, thread gauges, CMM, best fit, GUM

### 1. Introduction

Conventional measurement strategies for calibrating thread gauges only refer to specific lines in the direction of the rotary axis and in special axial sections according to the EURAMET cg-10 guidelines [1]. This procedure does not yield reliable statements regarding the pairing of thread plugs and rings, i.e. the actual functional behaviour of such machine elements.

Moreover, the detection of several periodic deviations, which may result from pitch and reeling errors as well as eccentricity, has been neglected to date as standard measurement procedures are not capable of giving the required information.

The state of the art in industrial metrology offers advanced technology for gathering measured values with a length accuracy of less than 1  $\mu\text{m}$  in three dimensions. It is about time to take advantage of these technologies to improve the calibration of 3D measurement objects.

This paper deals with the application and measurement results of an evaluation of a continuous thread surface, which is described by a parametric representation.

### 2. Methodology

The determination of parameters is carried out by a non-linear  $L_2$ -approximation algorithm, which is designed for the evaluation of 3D datasets in a Cartesian coordinate system gathered by a CMM [2]. For the efficient and stable solution of  $L_2$ -approximation problems, different mathematical approaches, e.g. the Gauss-Newton method or the Levenberg-Marquardt method, are well known and applied in practice [3].

For the mathematical description of a thread surface, it is necessary to divide the flanks into two mathematically independent screw surfaces with a parametric representation:

$$S_1(u_1, v_1, p_1, a_1) = \begin{pmatrix} u_1 \cdot \cos(v_1) \\ u_1 \cdot \sin(v_1) \\ v_1 \cdot \frac{p_1}{2\pi} - u_1 \cdot \tan(a_1) \end{pmatrix} \quad (1)$$

$$S_2(u_2, v_2, p_2, a_2) = \begin{pmatrix} u_2 \cdot \cos(v_2) \\ u_2 \cdot \sin(v_2) \\ v_2 \cdot \frac{p_2}{2\pi} + u_2 \cdot \tan(a_2) \end{pmatrix} \quad (2)$$

Formulas (1) and (2) describe the upper and lower thread surfaces in their major axes. They include the geometrical parameters pitch  $p_i$  and flank angles  $a_i$ . The variables  $u_i$  and  $v_i$  specify the surface coordinates.

The result of the 3D best fit delivers the geometrical parameters and the parameters for the position and orientation of the surfaces in the workpiece coordinate system separately.

### 3. Measurements

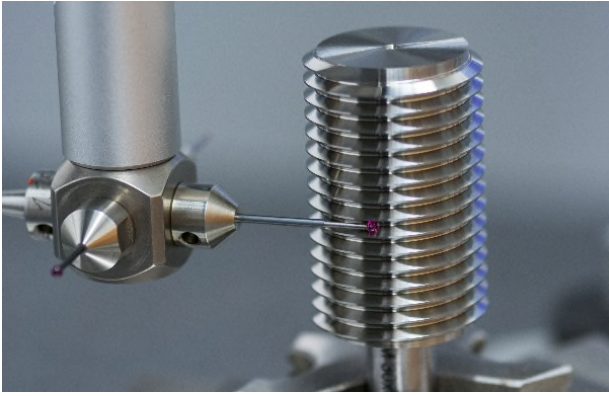
The first measurements were carried out on a set of eleven thread gauges with a nominal specification of M30x3 with thirteen full rotations. Every single gauge has a different modification of its nominal parameter values. The design drawing of the manufacturer shows combinations of modifications between the pitch, the angle and the geometry of the thread flank.

The measuring program for the CMM was only designed for the standard specification of M30x3. The present modification of the gauge is thus unknown for the CMM. This procedure reflects a common application in a calibration laboratory.

For a sound evaluation of the achieved results, two different precision CMMs were used at PTB:

- Zeiss UPMC 850 and
- Zeiss Prismo ultra.

Both CMMs are equipped with an RT-AB rotary table.



**Figure 1.** Probe in self-centring contact with a thread gauge

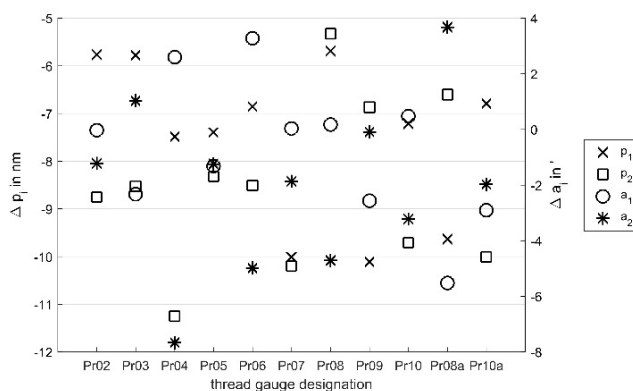
Figure 1 illustrates the applied measurement strategy with self-centring contact between the two flanks. During the whole data acquisition process of the stylus centre path along the programmed helix, the stylus stayed in self-centring contact with a fixed sensor axis tangential to the points of contact with the geometry. The stylus radius correction from the centre points to the contact points deals with the nominal vectors of the theoretical surfaces from formulas (1), (2) and the calibrated diameter value of the spheres. Therefore, the mentioned strategy allows a separated and independent best fit of the surfaces  $S_1$  and  $S_2$ .

In order to achieve a reconstruction of the flanks with the measuring strategy mentioned above, it is necessary to use at least two different probing spheres, gathering points at two cylinders with different diameters. For a more stable evaluation, we chose four different styli arranged as a star configuration. In total, about 40,000 points were probed in scanning mode per single measurement. Ten repetition measurements have been performed for the statistical analysis.

#### 4. Results

Figure 2 illustrates the comparison measurement results from two different CMMs. It shows the differences in terms of the geometrical parameters  $p_i$  and  $a_i$  as defined by equations (1) and (2) computed by the best fit algorithm.

The vertical axis in figure 2 is split into two scales: one for pitch  $p_i$  in nm on the left, and one on the right for the flank angle  $a_i$  in  $^\circ$ . The horizontal axis is labelled with the designation of the thread gauges. The flanks of the thread gauges from Pr02 to Pr05 are designed with straight lines, whereas the rest of the gauges are designed with concave or convex geometry.



**Figure 2.** Comparison of measurement results between UPMC 850 and Prismo ultra each with an RT-AB

The differences of pitch in a range of several nanometres are very small. This is, of course, due to the large number of measurement points used for the Gauss fit. The measurement of the flank angle proved to be more complicated and sensitive in the algorithm. One major cause is the short length of the flank in the radial direction which is only a small area in relation to the whole mathematical description.

Moreover, the gauges with straight line geometries have a smaller spreading than the gauges with concave and convex geometries. One possible reason might be that the implemented parametric representation of the flank geometry deals only with a straight line. Furthermore, the nominal form modification has not been taken into account for the gauges from Pr06 to Pr10a.

**Table 1.** Estimated standard uncertainties for geometrical parameters according to GUM type A and Monte Carlo simulation

Standard uncertainty of geometrical parameters	GUM type A	Monte Carlo n = 200
$u(p_1)$ in nm	2.4196	1.5591
$u(p_2)$ in nm	8.7969	1.5796
$u(a_1)$ in $^\circ$	0.0881	0.0592
$u(a_2)$ in $^\circ$	0.3184	0.0615

Table 1 shows a comparison of the estimated uncertainties for the best fit algorithm by two different methods in the theory of errors. The basis of these investigations is the measurement of thread gauge Pr03. The estimated uncertainties in table 1 represent only one component of an overall measurement budget based on the above-mentioned algorithm.

The method according to GUM type A deals with the evaluation of the distribution of the final residual.

For the Monte Carlo simulation, it was necessary to evaluate the distribution of the measured points in the x-, y-, and z-axes out of ten repeated measurements. These characteristic values were the input for the Monte Carlo simulation with a sample size  $n = 200$ .

Comparing the estimated values in table 1, there seems to be one irregularity for  $u(p_2)$  and  $u(a_2)$  in the GUM type A column. This irregularity is reducible to the algorithm, which assumes a normal distribution for the final residuals. A closer look at the distribution of the final residuals reveals that this assumption has been violated. Residuals with other than normal distribution refer to a systematic error in the measurement procedure.

#### 5. Conclusion and outlook

This paper gives an overview of the results for thread gauges achieved with two different CMMs and the application of an integral 3D approach to evaluation. The results in section 5 show that the applied measurement strategy includes systematic errors. These errors have to be determined and reduced. Additionally, the percentage of estimated uncertainties with the applied algorithm are small enough for a highly accurate calibration of thread gauges at PTB.

#### References

- [1] EURAMET, cg-10, v2.1 2012 Determination of pitch diameter of parallel thread gauges by mechanical probing
- [2] Sourlier D 1995 Three dimensional feature independent bestfit in coordinate metrology, Diss. ETH No. 11319
- [3] Madsen K, Nielsen H B and Tingleff O 2004 Methods for non-linear least squares problems, 2<sup>nd</sup> edition, IMM Technical University of Denmark