Integrated design of controller and JDC by a constrained H₂ optimization algorithm

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Abstract

Linear direct feed drive is an excellent choice to meet the requirement of high speed and ultra-high precision due to the simplicity of its mechanical structure. One major disadvantage is the jerk dependent reaction force on the secondary part of the linear motor. Jerk decoupling technique is a popular approach to decouple the reaction force from the machine frame. To systematically take care of both the tracking error and the jerk induced to the machine frame, this work takes an integrated approach to design the optimal JDC and the position controller of the feed drive. First, the mechatronics design problem is converted to a constrained H₂ optimization problem. To efficiently solve it, we propose an algorithm based on direct computation of projected gradient matrix and linear searching of step size. From the simulation results, we can conclude optimal parameters obtained by the proposed algorithm efficiently decouple the jerk induced to the machine frame and achieve good tracking performance of the primary part.

Keywords—linear direct feed drive, jerk-decoupling, constrained optimization, H₂ optimal control.

1. Introduction

Linear direct feed drives are widely used in high performance equipment and machine tools. However, a counter force from the secondary part of the linear motor induces jerk to the machine frame and causes residual vibration. Jerk-decoupling cartridge (JDC), i.e. spring-damper system, can be implemented between the secondary part of the linear motor and the machine frame as a buffer to decouple the reaction force [1][2]. Design targets include the tracking error and the jerk induced to the machine frame, this work takes an integrated approach to efficiently optimize controller and JDC parameters. The paper is organized as follows: section 2 presents the H₂ optimization problem formulation. To solve the problem, a gradient-based algorithm is presented in section 3. Section 4 showcases an example with simulation results. Lastly, section 5 concludes the paper with salient points.

2. Parameter optimization via H₂ formulation

A linear direct drive mounted on a machine frame through the JDC is shown in Fig. 1, and the modeling of the system is illustrated in Fig. 2. The original state space model of the system is expressed as \( x = Ax + Bu + w \), where \( x = [y_1, y_2, y_3, \dot{y}_1, \dot{y}_2, \dot{y}_3] \). We take the derivative of state space model and augment \( y_1 + y_3 \) as a state variable, the new state space model is given by \( \dot{x} = \ddot{A}x + B\ddot{u} + B_1\ddot{w} \), where \( \ddot{w} \) is assumed to be a unit impulse response. The \( \ddot{p} = [p_1 + y_3, p_2 + \dot{y}_3, p_3 + \ddot{y}_3] \) from a 3rd order autonomous trajectory generator is constructed as \( \ddot{p} = A_2\ddot{p} \), \( y_{1d} = c_d\ddot{p} \) and the reference trajectory is augmented with the state space model, which can be expressed as \( \ddot{x} = \dddot{A}x + B\dddot{u} + B_1\dddot{w} \), where \( \dddot{x} = [\dddot{p}, \dddot{y}] \), \( \dddot{A} = \begin{bmatrix} A_2 & 0 \\ 0 & \dddot{A} \end{bmatrix}, B_2 = [0, B_1], B_1 = [0, B_3], \dddot{u} = \ddot{u}, \dddot{w} = \dddot{w} \).

The objective is to design parameters in the controller and the JDC concurrently to minimize the force chattering induced to the machine frame \( F_2 \), meanwhile taking care of other design factors, i.e. tracking error of the primary part \( y_1 \), where \( y_1 = y_1 - p_1 \), the velocity of the secondary part \( y_2 \) and the machine frame \( y_3 \), as well as the chattering of the control voltage \( u_{in} \). Thus, the objective function is defined as \( J = \int_0^\infty z^Tz \, dt \), where \( z \) is the controlled output expressed as \( z^T = [y_1 - p_1, y_2, y_3, \dddot{u}_{in}, F_2] \). Equivalently, \( z \) can be expressed as \( z = C\dddot{x} + D\dddot{u} \). Now, we introduce one transfer function \( H(s) = (C + DK)[S I - (A + B_2K)]^{-1}B_1 \). The optimization problem is equivalent to minimize the \( H_2 \) norm of \( H(s) \) from \( \dddot{w} \) to \( z \) and the functional cost is expressed as \( J = \|H\|_2^2 = \text{Trace}[(C + DK)W(C + DK)^T] = \text{Trace}(B_1^T W_1 B_1) \), where \( W_1 \) and \( W_0 \) are controllability and observability Gramians of the closed-loop system.

![Figure 1. Linear direct feed drive with JDC configuration](image)

The decentralized composite controller is given by \( v = \dddot{u} = K\dddot{x} \), where the gain matrix \( K \) is structured as \( K = \begin{bmatrix} k_{x1} & k_{x2} & k_{x3} & -k_i & -k_p & -k_d \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & k_{x2} & k_{x3} \end{bmatrix} \begin{bmatrix} k_{d1} & k_{d2} & k_{d3} & k_{d4} \end{bmatrix} \).

From \( v \), we can restore the original controller in the actual system \( \dddot{u} \) by integration. Notice that the gain matrix must be...
sparse-restricted with certain constraints: (a). To make sure control of states in primary parts does not use the feedback from the secondary part, and the mechanical properties of JDC does not use the feedback from the primary part, thus, certain values in the gain matrix are set to zero; (b). The jerk-decoupling force \( F_2 \) is set to be \( F_2 = k_2y_2 + b_2\dot{y}_2 \). In this case, \( k_{d1} = -k_{d3} = k_2, k_{d4} = -k_{d5} = b_2; \) (c). The use of feedback from the machine frame, i.e. \( y_3, \dot{y}_3 \) and \( \ddot{y}_3 \) can be avoided by setting \( k_{x1} + k_{x4} = 0, k_{x2} + k_{x5} + k_{x7} = 0, k_{x3} + k_{x6} + k_{x9} = 0 \). For the primary part of the linear motor, a 2-DOF controller are implemented, where \( k_{x1}, k_{x2} \) and \( k_{x3} \) are feed-forward controller gains; \( k_p, k_d \) and \( k_{d3} \) correspond to the integral gain, the proportional gain and the derivative gain of the PID-type controller. For the secondary part, JDC is used for regulating purpose, where \( k_{d4} \) and \( k_{d5} \) are the stiffness \( k_2 \) and the damping coefficient \( b_2 \) of the JDC respectively.

![Figure 2](image-url) Modelling of feed drive connected to a machine frame

3. Constrained \( H_2 \) optimization algorithm

Because of the constrained feedback structure of the gain matrix, no standard closed-form solution is available to solve the optimization problem. However, it can be solved using a gradient-based constrained optimization algorithm [3][4].

Theorem 1. For the \( H_2 \) optimization problem in the infinite horizon, the matrical gradient of the functional cost with respect to the gain matrix is given by

\[
\frac{df}{dk} = 2(D^T DK + B^T W_c) W_c
\]

The proposed optimization algorithm is as follows:

**Step 1:** Set \( i = 1 \) and initialize a stable gain \( K^1 \) that satisfies all the constraints; **Step 2:** Calculate the controllability and observability Gramians \( W_c \) and \( W_o \); **Step 3:** Calculate matricial gradient using Theorem 1; **Step 4:** Determine the projected gradient matrix onto structural constraints and denote it by \( D \).

**Step 5:** Do the linear searching and find the optimal step size \( \alpha \) to minimize the cost after each iteration, written as \( \min_{\alpha} (K - \alpha D) \); **Step 6:** Update the gain matrix as \( K^{i+1} = K^i - \alpha D \); **Step 7:** Go back to Step 2 to continue the iterations until the functional cost converges.

4. Optimization and simulation

The optimization and simulation are done based on the ML50-2E-NC-MP linear motor. Initialize the gain matrix in the proposed constrained optimization algorithm as

\[
K = \begin{bmatrix}
    5 & 5 & 5 & -5 & -50 & -10 & 10 & 0 & 45 & 5 \\
    0 & 0 & 0 & 0 & 0 & 0 & 100 & 50 & -100 & -50
\end{bmatrix}
\]

which defines a stable decentralized configuration of gain matrix with a cost of 287.1. After 50 iterations, the optimized gain matrix is updated as

\[
K = \begin{bmatrix}
    183 & 45 & 33 & -183 & -46 & -34 & 0 & 0 & -1.4 & 1.8 \\
    0 & 0 & 0 & 0 & 0 & 1.93 & -1 & -9.3
\end{bmatrix}
\]

with a cost of 0.61. Fig. 3 shows the functional cost and the norm of the projected gradient matrix within the 50 iterations. We can observe that \( \|D\| \) is reduced from 42 to 0.008.

![Figure 3](image-url) Functional cost and norm of D within 50 iterations

![Figure 4](image-url) Simulation results of tracking error and chattering of jerk-decoupling force

5. Conclusion

This paper presents an optimal integrated design approach in the high-speed linear direct drive. The mechatronics system design problem formulation is done in non-inertia frame because all the encoder readings are relative positions with respect to the machine frame. Then, the mechatronics design problem is converted to a constrained \( H_2 \) optimization problem. As there is no closed-form solution to constrained \( H_2 \) problem, a gradient-based algorithm is proposed and optimal controller and JDC parameters are obtained. From the comparison of simulation results, we can conclude optimized parameters by the proposed algorithm efficiently decouples the jerk induced and other design requirement are met as well.

References


