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A local tool path smoothening scheme for micromachining

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Abstract
Linear and circular representations are widely used to define tool paths, however, the tangency discontinuity between the linear and circular segments leads to large fluctuations in velocity and acceleration, as a result, the machining accuracy and efficiency are degraded. It becomes the key problem in some micromachining situations where the quality of freeform surfaces is critical, such as moulds and knee implants, etc. This research aims to develop a local tool path smoothening scheme to achieve C^2 continuity at the transition positions. This scheme applies to sections consisting of high density of short segments. These segments will be approximated by cubic B-splines. The approximation is carried out within the specific error tolerance. High frequency energy to be injected into the servo loop control system is greatly reduced by the C^2 continuity. The proposed scheme is feasible to be implemented in real-time microcontrollers due to the computational efficiency and reliability of B-spline algorithms.

Tool path smoothening; B-spline; Micromachining

1. Introduction

The conventional method to machine freeform surfaces relies on programs generated by commercial computer-aided manufacturing (CAM) software. To achieve the desired accuracy, the program usually consists of a large volume of short line and circular segments. The velocity discontinuity at each transition point between two segments will lead to unacceptable jerk. The common method used by commercial computer numerical controllers (CNC) is to decelerate first when approaching the transition point, and then accelerate to the specified feedrate at the start of the next segment. The frequent acceleration and deceleration will cause machine vibration that will degrade the surface finish and machining accuracy. Furthermore, the machining time is increased significantly due to the feedrate fluctuation. It is becoming an increasingly urgent issue to be solved in micromachining realm.

Siemens has implemented a corner rounding technique in its CNCs [1], the C^2 continuity is achieved at the transition point by inserting a spline between two segments. Similarly, Xavier et al. [2] developed corner rounding techniques for 5-axis milling based on cubic B-spline. However, the corner rounding makes little difference when dealing with high density of short segments, because the connection splines are too short to create smooth transitions. In recent years, Non-uniform Rational B-Spline (NURBS) and some other interpolators have been a hot research topic [3,4]. Unlike the corner rounding technique that still depends on the linear and circular interpolations, those interpolators are completely new interpolation methods that avoid the discontinuity problem. However, the lack of CAM software and CNC support is a severe limitation, it is also not attractive to industry engineers, because it is not as intuitive as linear and circular interpolations.

This paper presents a scheme for smoothening the tool path with high density of short segments, the scheme uses cubic B-splines to approximate the points, the approximation is an iterative process to limit the curve within the tolerance.

The rest of the paper is organised as follows. A brief introduction of B-spline representation is given in section 2, then the approximation algorithm is described in section 3, finally this study is concluded in section 4.

2. B-spline representation for tool path

2.1. B-spline representation

B-spline provides a relatively simple mathematical form capable of representing freeform curves. A kth-degree B-spline curve is given by

$$\mathrm{P(t)} = \sum_{i=0}^{n} d_i N_{i,k}(t) \tag{1}$$

Where $\mathrm{P(t)}$ is the position vector along the curve as a function of parameter $\mathrm{t} (\mathrm{a} \le \mathrm{t} \le \mathrm{b})$. $d_i (i = 0, 1 \ldots, n)$ are the control points, connecting the control points in sequence yields a control polygon. $N_{i,k}(t) (i = 0,1, \ldots, n)$ are the kth-degree B-spline basis functions, they are piecewise polynomials defined on knot vector U, $\mathrm{U} = \{u_0, u_1, \ldots, u_{n+k+1}\}$ is a nondecreasing sequence of real numbers. The basis functions are given by

$$\begin{cases} N_{i,0}(t) = \begin{cases} 1, if\ u_i \le t \le u_{i+1} \\ 0, Otherwise \end{cases} \\ N_{i,k} = \frac{t-u_i}{u_{i+k}-u_i} N_{i,k-1}(t) + \frac{u_{i+k+1}-t}{u_{i+k+1}-u_{i+1}} N_{i+1,k-1}(t) \end{cases} \tag{2}$$

In Equation (2), The convention $\frac{0}{0} = 0$ is adopted. There are a number of properties of B-splines that make the B-splines apt to represent freeform curves [5].

2.2. C^2 continuity of tool path

The smoothness of a parametric curve can be evaluated by the degree of parametric continuity, which depends on the continuity of derivatives. If two curves are joined together, then the zeroth-order of derivatives at the join point are equal, which implies C^0 continuity. Apparently a tool path has C^0 continuity everywhere. If the tangent vectors (first-order derivatives) are identical (both direction and magnitude), then the resulting curve is C^1 continuous at the join point. For tool paths using linear and circular representations, C^1 continuity is usually lost, i.e., the velocity of tool has an instantaneous change (both direction and magnitude or one of them) at the join point, this is highly undesirable for the aforementioned reasons.

If a tool path has C^2 continuity, viz. it has continuous second-order derivatives, then both velocity and acceleration of the tool are continuous, jerk limitation is achieved. For many applications, C^1 continuity is adequate, but for high-speed, high-precision micromachining, at least C^2 continuity is required to satisfy the increasingly demanding accuracy requirement.

Piegl and Tiller [5] proved that if C^r continuity is desired for a curve, then the chosen degree k must satisfy

$$k \geq r + 1 \quad (3)$$

The precondition of Equation (3) is that the multiplicities of interior knots are not greater than 1. In this application, degree 3 is chosen for the B-splines to satisfy the C^2 continuity requirement, as well as to minimize the computation intensity.

3. Tool path approximation

Suppose a part of tool path is given by a set of line segments, the length of each segment is shorter than the predefined threshold L. Then end points are extracted from these segments, the resulting set of points $\{P_i\}, i = 0, \dots, n$, are the points to be approximated. Four basic steps are required to perform the approximation.

3.1. Choose parameters t_i

The most widely used method to choose t_k is chord length method [5]. It gives a "good" parameterization to the curve.

$$D = \sum_{i=1}^{n} |P_i - P_{i-1}| \quad (4)$$

$$t_i = t_{i-1} + \frac{|P_i - P_{i-1}|}{D} \quad i = 1, \dots, n-1 \quad (5)$$

Where D is the total cord length. $t_0 = 0, t_n = 1$.

3.2. Choose knot vector U

Piegl and Tiller [5] recommended using averaging technique to choose the knot vector, the resulting knot vector U reflects the distribution of parameters t_i.

$$u_{j+k} = \frac{1}{k} \sum_{i=j}^{j+k-1} t_i \quad j = 1, \dots, n-k \quad (6)$$

The other knots are: $u_0 = \cdots = u_k = 0$, $u_{n+1} = \cdots = u_{n+k+1} = 1$.

3.3. Compute unknown control points d_i

The required number of control points depends on the specified tolerance E. In this application, the initial number of control points starts from 4. The required control points are the only unknowns now, so they can be computed by solving the linear least square problem.

3.4. Check the deviations e_i

If the maximum deviation from the approximated points to the fitting curve is within tolerance E, then the curve is the desired one. Or the number of control points should increase by one until the deviation condition is satisfied.

There is a trade-off when choosing tolerance E, the smaller E is, the more accurate the curve is, but it is at the expense of longer computing time and the sharper corners. Figure 1 illustrates the iterative process.

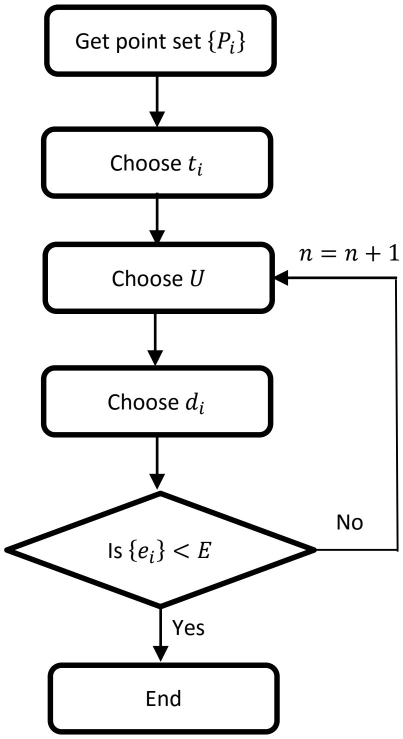

Figure 1. The iterative process

4. Conclusion

The paper presents a local tool path approximation method, which is based on cubic B-splines. This method applies to the tool path consists of high density of short segments. C^2 continuity is achieved within the approximated curve, which avoids the feedrate fluctuation problem comes with linear and circular interpolations. The approximation algorithms are discussed, if they are implemented in CNCs, then the currently widely used machining programming method keeps the same.

The connections between the approximated curves and long line segments should be solved in the future work to make the scheme applies to the global tool path.

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