

## Validation of pre-sliding friction models for ultra-high precision applications

Ervin Kamenar<sup>1</sup> and Saša Zelenika<sup>1</sup>

<sup>1</sup> University of Rijeka, Faculty of Engineering & Centre for Micro and Nano Sciences and Technologies, Vukovarska 58, 51000 Rijeka, CROATIA

[ekamenar@riteh.hr](mailto:ekamenar@riteh.hr)

### Abstract

Friction is one of the main disturbances in precision positioning and, in the pre-sliding motion regime, it is characterised by an elasto-plastic nonlinear hysteretic behaviour with a marked variability. Pre-sliding friction, modelled via the state-of-the-art Generalised Maxwell Slip (GMS) and the Hsieh model (HM), is experimentally assessed in this work with the aim of developing suitable control typologies aimed at compensating its effects. The models are hence compared in terms of the complexity of identifying their parameters, simulating their dynamic responses and implementing them in real-time systems. It is thus shown that the GMS model is readily identified and comprises also the sliding motion regime, whereas HM is difficult to implement and requires a separate sliding friction model with a corresponding switching function.

Pre-sliding friction, GMS & Hsieh models, identification of parameters, modelling, validation

### 1. Introduction

In nanometric positioning, one of the main disturbances is friction that can induce steady-state errors in point-to-point positioning, tracking errors, large settling times, stick-slip etc. What is more, recently it has been shown that ultra-high precision positioning often happens in the pre-sliding motion regime where friction shows a clear qualitative trend, which has, however, a marked variability. With the aim of developing robust control typologies for ultra-high precision positioning devices, several pre-sliding friction models have thus been recently proposed [1-2]. The pre-sliding frictional behaviour is experimentally assessed in this work so as to identify the parameters of two state-of-the-art grey-box physical models: the Generalised Maxwell Slip (GMS) [1] and the Hsieh model (HM) [3]. These models take into account the elasto-plastic nonlinear force-displacement characteristic of pre-sliding, with its hysteretic and non-local memory effects (Fig. 1). Results of the experimental measurements allow validating the physical foundation of the considered models. The models are compared also in terms of the level of complexity in identifying the respective parameters, of the simulation of their dynamic responses as well as of their implementation in control systems.

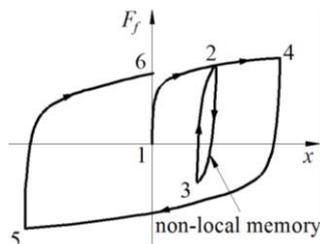


Figure 1. Pre-sliding friction behaviour.

### 2. Friction models

Pre-sliding friction occurs for tangential forces smaller than breakaway and is characterized by micro-slip displacements with a non-linear stochastic behaviour that can amount up to several hundred micrometres (Fig. 1). It can be modelled by

using different physical models; two most recent and comprehensive ones are considered in this work.

#### 2.1. GMS model

The GMS friction model is based on  $N$  massless Maxwell elasto-slip blocks connected in parallel that have all the same input – velocity  $v$  and one output – the friction force  $F_i$  acting on the  $i$ -th block (Fig. 2). Two states of either hysteresis with non-local memory in pre-sliding (where velocity  $v$  is the derivative of the state variable  $z$ , which corresponds to the average deflection of surface asperities) and slip with frictional lag, determine hence the behaviour of each block. This behaviour depends also on the stiffness  $k_i$  of each block and its force limit  $W_i$  when the block starts slipping, entering thus a velocity-weakening curve bounded on the lower end by Coulomb friction [1]. The parameters of the model can be determined by the piecewise approximation of the experimentally obtained friction force  $F_f$  versus displacement  $x$  curve [4].

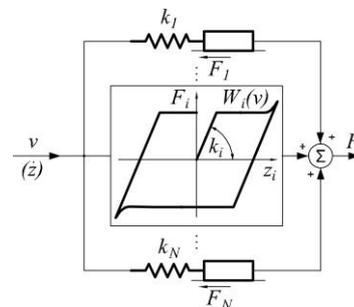


Figure 2. Schematic representation of the GMS slip blocks.

#### 2.2. HM

The HM friction model is composed of two modules: a plastic and a nonlinear spring one (Fig. 3). The plastic module is mounted in series with the spring module and it comprises plastic deformations that demonstrate creep (continuous deformation under constant load) and work hardening (close-to-isotropic deformation whose rate decreases and then stops; work hardening accumulated in one direction applies also to the other). The nonlinear elastic module demonstrates conversely a hysteresis loop with memory and wipe-out effects. The respective spring element is mounted in parallel

with a viscous damper (with a coefficient  $C_v$ ), which takes into account energy dissipation. A pre-sliding friction force  $F_f$  will thus arise in both modules due to the action of an external force  $F$ , while the overall displacement  $x$  will include the deformation  $x_p$  of the plastic module and the displacement  $x_s$  of the spring module [3].

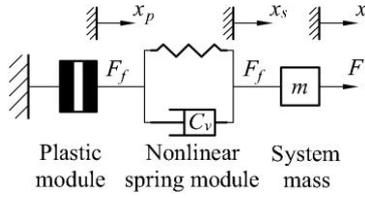


Figure 3. Schematic representation of Hsieh's friction model.

The plastic behaviour with its work hardening  $x_h$  is defined as:

$$\dot{x}_h = \begin{cases} \rho \left[ \frac{F_f^a}{\psi} - x_h \right], & \text{if } \left( \frac{F_f^a}{\psi} \right) > x_h \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$\dot{x}_p = \text{sgn}(F_f) \dot{x}_h$$

where  $a$ ,  $\psi$  and  $\rho$  are positive constants. On the other hand, the nonlinear spring module, depending on the direction of motion of the system of mass  $m$ , is defined as:

$$F_f = \begin{cases} k_{1s}(x_s - x_r) + \frac{k_{2s}}{\beta} (1 - e^{-\beta|x_s - x_r|}) + \sigma_r + C_v \dot{x}_s, & \text{if } x_s \geq x_r \\ k_{1s}(x_s - x_r) + \frac{k_{2s}}{\beta} (e^{-\beta|x_s - x_r|} - 1) + \sigma_r + C_v \dot{x}_s, & \text{if } x_s < x_r \end{cases} \quad (2)$$

where  $k_{1s}$  and  $k_{2s}$  are stiffness coefficients,  $\beta$  is a positive scalar, while  $\sigma_r$  and  $x_r$  are, respectively, the force and the displacement of the hysteresis loop appearing when the direction of motion is reversed (e.g. points 2 and 3 in Fig. 1) [3]. It can be thus inferred that HM comprises a large number of characteristic parameters.

### 3. Experimental identification and discussion

The pre-sliding behaviour of a linear stage is measured by using an elaborated experimental set-up. Tangential loading, applied via a micro-tensile machine, is increased in sequential steps initiated only when the system comes to an almost complete rest, since displacements on the nanometric level can be observed even after extended time periods. The resulting displacements are measured via a laser interferometric system. In order to validate the position and time variability of frictional effects, more than 50 experiments are conducted [4]. As shown in Fig. 4, pre-sliding displacements occur for forces up to  $\sim 0.9$  N and displacements amounting to  $\sim 30$   $\mu\text{m}$  (the respective dispersion of up to  $\sim \pm 15\%$  in repetitive measurements is not shown to enhance the clarity of the figure).

The obtained experimental data allow identifying the parameters of both the GMS and the HM friction model. The respective values are identified readily in the case of the GMS model via a simply piecewise linear approximation of the experimental curve [4]. On the other hand, the parameters of the HM are physically hard to interpret and are identified via additional cumbersome qualitative trial-and-error off-line procedures [3]. In fact, instead of estimating all parameters together as in the case of the GMS model [4], the motion corresponding to the characteristics of each of the cited modules is isolated and the associated parameters are estimated separately [3]. The resulting values of all the characteristic parameters of both frictional models are reported in table 1. These values are inserted in the Matlab/Simulink model of the pre-sliding behaviour of the studied linear stage and the respective responses are depicted

in Fig. 4. It is thus evident that both models allow describing well the pre-sliding frictional behaviour, but in the HM case this is achieved once more at the expense of a difficult implementation of the model. All this implies that HM is very difficult to use in control systems, considering also that it is limited to pre-sliding friction and requires a separate sliding friction model with a corresponding switching function, which makes it impractical for real-time controllers, especially those based on disturbance observers [2]. For this reason, the compensation of frictional disturbances on an actual high-precision positioning system will be achieved by modelling frictional effects via the GMS model that can encompass sliding friction as well [5].

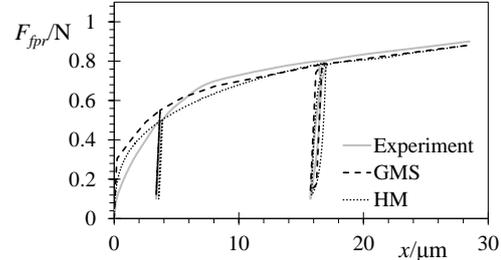


Figure 4. Comparison of experimental data with responses modelled via the GMS and HM friction models.

Table 1. Parameters of the GMS and HM friction models.

GMS		HM	
$k_i/\text{N}\cdot\text{m}^{-1}$	$\alpha_i$	$k_{1s}/\text{N}\cdot\text{m}^{-1}$	$k_{2s}/\text{N}\cdot\text{m}^{-1}$
454610	0.378	51540	2026400
111088	0.138	$C_v$	5000
45601	0.148	$\beta$	3216500
16402	0.177	$\rho$	500
6899	0.112	$\psi$	31200
4295	0.046	$a$	3.22

### 4. Conclusions

Two recently proposed pre-sliding friction models are analysed in this work, their characteristic parameters are experimentally assessed and their performances are compared. Based on this study, the following conclusions can be drawn:

- Although both models can describe well pre-sliding frictional behaviour with hysteretic and non-local memory effects, the numerous parameters of the HM are far more difficult to identify, physically hard to interpret and challenging to incorporate into simulation models.
- While the GMS friction model can encompass also the sliding behaviour without the need for a corresponding switching function, HM is limited to pre-sliding only.
- This implies that the usage of the GMS model in control systems aimed at compensating frictional disturbances is rather straight-forward, while the implementation of HM in real-time control systems is impractical, especially in case of controllers based on disturbance observers.

### Acknowledgements

The work was made possible by using the equipment funded via the ERDF project no. RC.2.2.06-0001: "Research Infrastructure for Campus-based Laboratories at the University of Rijeka (RISK)".

### References

- [1] Al-Bender F, Lampaert V and Swevers J 2005 *IEEE T. Automat. Contr.* **50**(11) 1883-7
- [2] Chih-Jer L, Her-Tereng Y and Yun-Cheng T 2013 *IEEE-ASME T. Mech.* **18**(4) 1385-96
- [3] Hsieh C and Pan Y C 2000 *Wear* **242**(1-2) 1-17
- [4] Zelenika S and Kamenar E 2016 *these proceedings*
- [5] Kamenar E and Zelenika S 2016 *these proceedings*