euspen's 16th International Conference & Exhibition, Nottingham, UK, May 2016
www.euspen.eu

Development of a geometrical error model for coordinate measurements made by industrial X-ray computed tomography

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Abstract
X-ray computed tomography (CT) has been recognized as an effective tool for non-destructive testing of manufactured parts. The technology is particularly promising for dimensional quality control, as it provides users with the ability to perform coordinate measurement of both internal and external features. The application of CT for industrial inspection is recent and protocols for establishing measurement assurance, *i.e.* assessing measurement uncertainty, are not yet available. The research presented here is focused on the development of a model relating geometrical misalignments and error motions in the construction and operation of CT instruments to x, y, z coordinate errors in the tomographic volume. In particular, the model considers angular misalignments of the X-ray detector. An analytical derivation of the model is provided. The efficacy of the model is evaluated by comparing its output to observed coordinate errors from simulated data. The implications of such a geometrical error model to the eventual assessment of CT measurement uncertainty are presented.

Keywords: Computed tomography, dimensional metrology, geometrical modelling

1. Introduction

X-ray computed tomography is increasingly applied for dimensional inspection of industrial products [1]. The recent emergence of this measuring technology explains the lack of protocols for evaluating its performance. Measurement traceability is desired as it provides users with confidence in the accuracy of their measurements. A critical step in establishing traceability is the assessment of measurement uncertainty. According to the GUM method [2], uncertainty can be evaluated by characterizing each source of error (both systematic and random) in the measurement procedure and propagating its quantity to an uncertainty in the final measurement result.

In this study, systematic errors in volumetric measurements due to angular misalignments of the flat panel detector are investigated. These misalignments are parameterized and included in a geometrical error model to estimate final *x, y, z* errors in CT measurements. Preliminary results comparing the output of the error model to observed volumetric deviations in simulated data are presented and limitations are discussed.

2. Cone-beam CT geometry

A widely accepted method for tomographic reconstruction of volumes acquired by cone-beam CT is FDK filtered backprojection [3]. While individual reconstruction algorithms may differ in their implementation, the general concept of cone-beam CT is the same. Two-dimensional X-ray intensity images (radiographs) are acquired at various angular positions of the measurement volume. The measured intensity at each pixel on all radiographs is backprojected through the linear path of the X-ray from the source to the pixel coordinate in three-dimensional space. X-ray path intersections in the measurement volume are used together with the measured X-ray intensity to reconstruct the 3D volumetric attenuation map. Accurate reconstruction of the volumetric (voxel-based) attenuation map is dependent on accurate knowledge of the CT imaging geometry, *i.e.* relative position and orientation of X-ray source, rotation axis, and detector pixels, at all rotation positions of the measurement volume.

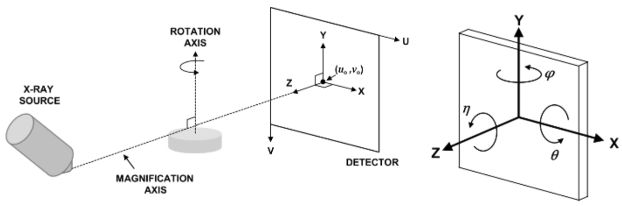

Figure 1. Left: Ideal cone-beam CT geometry. Right: Angular misalignments of the flat-panel detector can be parameterized as three rotation angles.

An aligned cone-beam CT system at a fixed rotation position of the measurement volume is depicted in figure 1, left. Typical cone-beam CT systems are considered aligned when the following conditions are met [4]: (a) The magnification axis, *i.e.* the line normal to the detector and intersecting the X-ray source, is centred on the detector; (b) the rotation axis is orthogonal to and intersects the magnification axis; and (c) the columns of the detector are parallel to the rotation axis. The distances along the magnification axis between X-ray source and rotation axis, *SRD*, and between X-ray source and detector, *SDD*, are assumed accurately known.

In this study, angular misalignments of the flat panel detector are parameterized. Angular misalignments can be described by three extrinsic, right-handed Euler rotation angles (θ, φ, η) about the central row, the central column, and the normal at the centre of the detector, respectively (figure 1, right). Other conventions for applying rotations to the detector are possible.

3. Methodology

The voxel-based volumetric error model consists of three steps: (1) forward projection of 3D voxel centre coordinates onto the ideal 2D detector pixel coordinates, (2) backprojection of 2D radiographic binning errors into 3D grey value binning errors, and (3) rotation of the backprojected 3D binning errors to $\alpha = 0°$ orientation.

Each voxel in the measurement volume is identified by three indices i, j, and k, denoting its position along the X, Y, and Z axes, respectively. In the first step, the centre coordinate $(x, y, z)_{\mathrm{ijk}}$ of each voxel is forward projected onto an ideal detector plane and the corresponding pixel coordinate $(u, v)_{\mathrm{ijk}}$ is evaluated.

The X-ray intensity recorded by pixel coordinate (u, v) in the ideal detector will be recorded by pixel coordinate (u_r, v_r) in the (θ, φ, η) misaligned detector. Radiographic errors for each pixel in the ideal detector are given by the difference in pixel coordinates

$$\mathrm{d}u(u, \theta, \varphi, \eta) = u_r - u \text{ and } \mathrm{d}v(v, \theta, \varphi, \eta) = v_r - v$$

The second step consists of backprojecting the radiographic errors $\mathrm{d}u$ and $\mathrm{d}v$ to the corresponding voxel as volumetric grey value binning errors $\mathrm{d}x$ and $\mathrm{d}y$. The first and second steps are repeated for incremented Y rotations α of the voxel space. Backprojected $\mathrm{d}x$ and $\mathrm{d}y$ values for $\alpha \neq 0°$ are rotated about an axis parallel to the Y axis and centred on the corresponding voxel centre to $\mathrm{d}x$, $\mathrm{d}y$, and $\mathrm{d}z$ at the $\alpha = 0°$ orientation.

For each voxel in the modelled volume, there is a set of N binning errors $\{\mathrm{d}x, \mathrm{d}y, \mathrm{d}z\}_N$. The mean of each binning error component $\overline{\mathrm{d}x}$, $\overline{\mathrm{d}y}$, $\overline{\mathrm{d}z}$ is evaluated for all voxels.

4. Results

The output of the volumetric model is compared to observed centroid-to-centroid distance errors in simulated scans of a test object (figure 2) in the presence of equivalent detector misalignments. The modelled $\overline{\mathrm{d}x}$, $\overline{\mathrm{d}y}$, $\overline{\mathrm{d}z}$ values for the voxels corresponding to the sphere positions in the ideal CT scan are used in calculating sphere-to-sphere distance errors. Preliminary comparisons indicate that the model captures systematic behaviours in the reconstructed volume. The plots in figure 3 compare the modelled distance errors to observed centroid-to-centroid deviations between spheres in the simulated volume for some representative detector rotations. The X-axis corresponds to each centroid-to-centroid segment from the central sphere to all other spheres in the volume. The scaling of the modelled dx and dz components provided an improvement in the fit between modelled and observed centroid-to-centroid deviations. The presence and magnitude of such a scaling factor is the topic of further investigation.

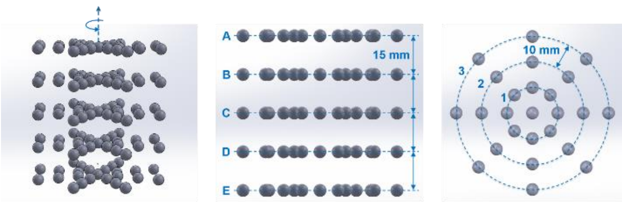

Figure 2. Simulated test object for evaluating dimensional distortions in measurement volume.

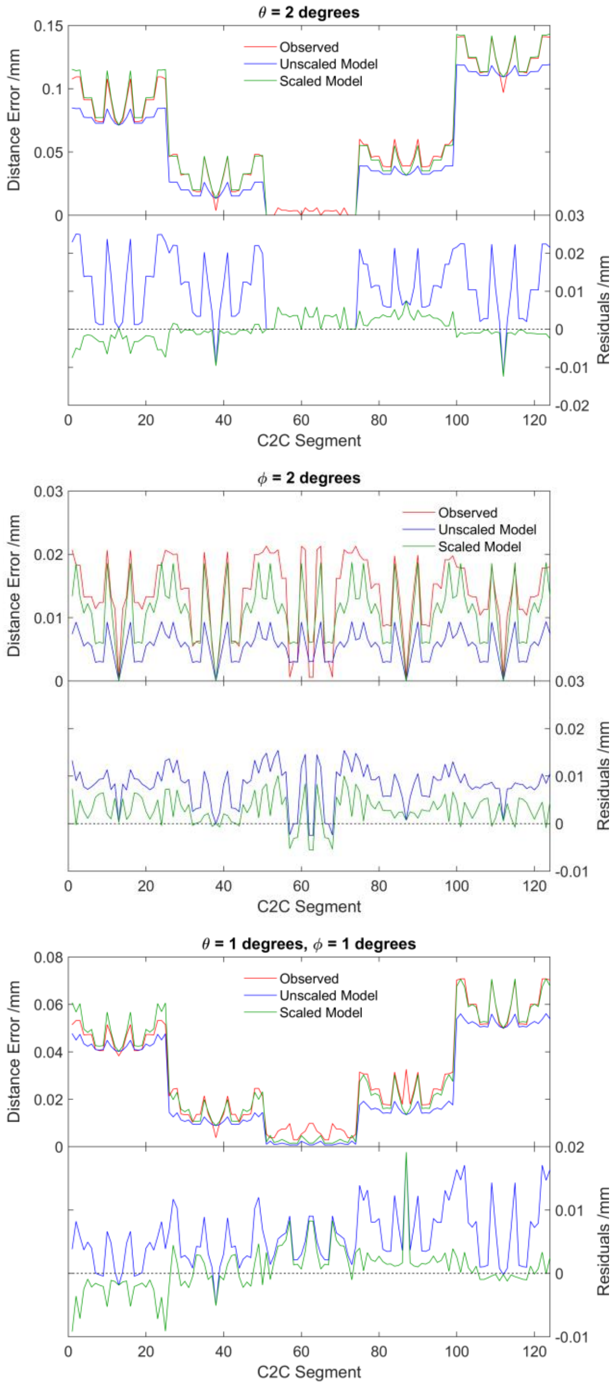

Figure 3. Comparisons to simulated data indicate that systematic behaviours are captured by the volumetric error model. Scaling the modelled dx and dz components significantly reduced the residuals.

Acknowledgments

This research was funded by the European Union's FP7 Framework Programme under the INTERAQ*CT* project, grant number 607817.

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