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## A novel method based on Bayesian regularized artificial neural networks for measurement uncertainty evaluation

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### Abstract

Coordinate measuring machines (CMMs) are complex measuring systems that are widely used in manufacturing industry for form, size, position, and orientation assessment. In essence, these systems collect a set of individual data points that in practice is often a relatively small sample of an object. Their software then processes these points in order to produce a geometric result or to establish a local coordinate system from datum features. The subject of CMM evaluation is a broad and multifaceted one. This paper is concerned with the uncertainty in the coordinates of each point within the measuring volume of the CMM. Therefore, a novel method for measurement uncertainty evaluation using limited-size data sets is conceived and developed. The proposed method is based on a Bayesian regularized artificial neural network (BRANN) model consisting of three inputs and one output. The inputs are: the nominal coordinates; the ambient temperature; and the temperature of the workpiece. The output is the measured (actual) coordinates. An algorithm is developed and implemented before training the BRANN in order to map each nominal coordinate associated with the other inputs to the target coordinate. For validation the model is trained using a relatively small sample size of ten data sets to predict the variability of a larger sample size of ninety data sets. The calculated uncertainty is improved by more than 80% using the predicted variability compared to the uncertainty from the limited sample data set.

Keywords: uncertainty of measurement, Bayesian regularized artificial neural network (BRANN), coordinate measuring machine (CMM)

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### 1. Introduction

Coordinate measuring machines (CMMs) are used extensively in manufacturing industry to carry out an inspection with high accuracy. Even though they only measure individual points in space, they are extremely flexible. Their flexibility comes from the software that processes these points in order to produce a geometric result or to establish a local coordinate system from datum features. Every point gathered by a CMM is expressed in terms of its  $x$ -,  $y$ -, and  $z$ - measured coordinates. Therefore, estimating the uncertainty of the coordinates of each measured point can help determine uncertainty contributors associated with a particular axis of the CMM [1] and enable very efficient implementations of geometric element best-fit algorithms [2].

For multivariate measurands such as a set of coordinates, uncertainties are evaluated in terms of variance matrices that can frequently be derived in terms of a measurement system model [3]. Nevertheless, this is not straightforward in the case of CMM measurement due to the complexity of the measurement process and the CMM itself [4]. As a result, the scope of the model is often limited to certain environmental and working conditions. In many applications, when no satisfactory mathematical model can be derived, artificial neural networks (ANNs) are a good alternative predictive modelling approach. ANNs learn from experience rather than by deterministic programming and they provide highly parallel, adaptive models trained only by input-output data. Also, they are able to generalize from given training data to unseen data. However, ANNs cannot be seen as a simple one-answer-fits-all solution, and in many cases misapplication of artificial intelligence techniques can lead to incorrect results, especially where the ANN model is poorly defined and perturbations are outside the scope of the training sample.

This paper is concerned with the uncertainty of measurement in the coordinates of each point within the CMM workzone. Therefore, a novel predictive model is developed for CMM performance evaluation using limited-size data sets. The proposed model is an ANN consisting of three inputs and one output. The ANN is trained by Bayesian regularization to improve network generalization. This approach, which is an improvement of back-propagation, uses statistical techniques so that the trained ANN can use the optimal number of parameters. Bayesian regularization provides better generalization performance than early stopping, especially for small data sets because it uses all the data; it does not require that a validation data set be separate from the training data set [5]. To attempt to realize such a model, all the inputs and the output are coded as vectors.

### 2. The proposed method

Consider that twenty individual data points representing the measured (actual)  $x$ -coordinates  $x_i$  ranged from 0 to 210 mm in this example are generated according to the model:

$$x_i = x_i^* + e_{a_i} + e_{w_i} + \varepsilon_i, \quad i \in I = \{1, \dots, 20\}, \quad (1)$$

where  $x_i^*$  is the nominal  $x$ -coordinates,  $e_{a_i}$  and  $e_{w_i}$  represent systematic effects associated with the ambient temperature and the workpiece temperature, respectively, and  $\varepsilon_i$  represents random effects. Suppose then that ten data sets including twenty actual  $x$ -coordinates each are generated according to this model with errors ranged from  $-10$  to  $12 \mu\text{m}$  (temperature values range from  $18$  to  $22^\circ\text{C}$ ). Consequently, each data point in each data set is highly correlated to an ambient temperature, a workpiece temperature, and a random, uncorrelated effect.

The multi-layer perceptron (MLP) network shown in Figure 1 consists of three input units, five hidden neurons and one output unit. The activation functions for both the hidden and the output layers are tan-sigmoid (tansig) transfer functions to provide the nonlinear characteristic. The three inputs of the network are the vector of nominal coordinates, the vector of ambient temperature data, and the vector of workpiece temperature data while the output (target) is the vector of actual coordinates (displacement). The nominal coordinates are used because they help the ANN to generalize for different measurement tasks across the CMM. An algorithm is developed and implemented before training the ANN in order to map the nominal coordinates associated with the other inputs to the target coordinates. The data has been normalized between -1 and 1, since the Bayesian regularization training algorithm generally works best when the ANN inputs and targets are scaled within that range [5].

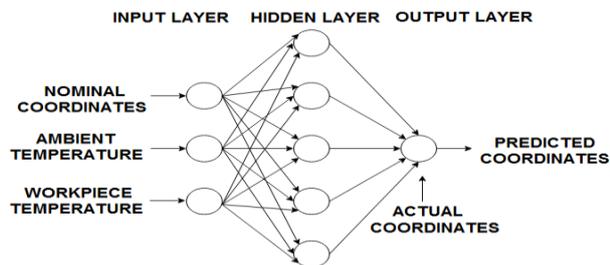


Figure 1. The MLP network with five hidden neurons.

By varying the simulations in MATLAB with different numbers of hidden neurons, four different models were developed. The first model consists of five hidden neurons, the second with ten, the third with twenty, and the fourth with forty. All the models were trained for a different number of epochs because the training process only needs to be implemented until the errors converge. In order to examine the performance of all the Bayesian regularized artificial neural network (BRANN) models on non-training data, another ninety data sets (testing sample) were generated. So, ten data sets were used for training and ninety data sets for testing.

The mean squared error (MSE) performance function was used to measure each network's performance. Table 1 shows the results obtained from all the developed models; the number of convergence epochs, the network's performance according to the mean of squared errors, and the percentage of improvement in the calculated uncertainty compared to the original data set.

Table 1. Performance of BRANN models.

Models	Epochs	MSE/mm	Calculated uncertainty improvement/%
1	2374	$4.65 \times 10^{-5}$	83.0
2	3744	$8.74 \times 10^{-6}$	66.5
3	3487	$1.68 \times 10^{-5}$	67.5
4	5000	$3.28 \times 10^{-5}$	66.0

Based on Table 1, it can be determined that the optimal solution in terms of the improvement in the calculated uncertainty over the sample statistics method is the first model. Figure 2 compares the standard uncertainties [6] simulated in the training sample with the uncertainties from the testing sample and the uncertainties from the predicted coordinates obtained by the four models for each sampling point.

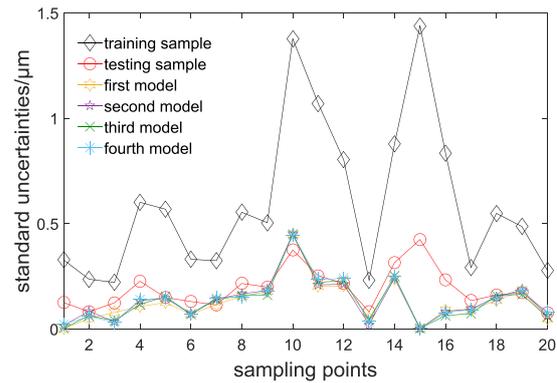


Figure 2. Standard uncertainties for each sampling point.

As can be seen from Figure 2, while sample statistics is misleading for small sample sizes, the proposed method is shown to be a good modelling approach to predict the variability associated with the point coordinates. In a similar way, the method can be applied to y- and z-coordinates.

### 3. Conclusions

The paper has been concerned with the point coordinate uncertainties. Therefore, an empirical method based on a BRANN model has been conceived and developed to predict the variability associated with the CMM coordinate data using limited-size data sets. A validation case study has shown that the calculated uncertainty is improved significantly using the predicted variability compared to the uncertainty from the limited sample data set. Therefore, the method can be applied to determine uncertainty sources associated with a particular axis of the CMM and increase the efficiency of geometric element best-fit algorithms implemented in coordinate data. Finally, the method could be extended beyond CMMs to include other measurement systems such as comparator gauges i.e. considering the difference between the temperature of the master part and the productions parts instead of the ambient temperature due to the principle of comparative measurement.

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