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Simulation study of a refined approach for specular surfaces reconstruction in deflectometry

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Abstract
Regularization is a necessary step in deflectometry method to solve the inherent ambiguity by incorporating additional information. When the detected surface is large, complicated or convex, regularization by stereo methods could only provide normal vector information within a small overlapping region. In this case, a refined approach for the estimation of normal fields of the non-overlapping surface points resulting from the overlapping data is considerable. In this paper, a new method based on the approximation that normal vector of one point is perpendicular to the vector of connecting points at either side is proposed to calculate the non-overlapping normal information. Then the surface shape is obtained by integrating the normal fields with least-square 2D integration method. Simulation work with noisy data on a convex specular surface verifies its feasibility and accuracy.

Keywords: specular surfaces reconstruction, finite difference method, deflectometry

1. Introduction

Deflectometry [1-2] is a method to measure local slope of specular surfaces through utilization of the deformation of a sample pattern after reflection from a tested surface, as shown in Figure 1(a). During the process of calculating gradient data, the inherent ambiguity needs to be resolved to get a unique relation between surface height and its normal vector. A general method to solve this ambiguity is by stereo vision [1-3], where the detected specular surface is observed under different angles by two CCD cameras, and the sample patterns are simultaneously reflected into the imaging systems via the detected surface. By correlating the normal vector fields induced by the two measurements, normal vectors within the overlapping field can be calculated. However, when the surface under test is quite large, complicated or convex, the overlapping region by stereo methods may be extremely small. In this case, full use of the information captured by each camera is made by estimation of the normal fields of the non-overlapping surface points resulting from the overlapping data. This paper proposes a new method to calculate the normal vectors of the non-overlapping region points, which is based on the approximation that the normal vector of one point is perpendicular to the vector connecting the points at either side. Section 2 gives a detailed description about the principle of this new method. A simulation is presented in section 3.

2. Principle

2.1. Model of deflectometry

The general geometric model is depicted in Figure 1(a), where we assume the detected regular surface can be expressed by an explicit function $F(x,y,z)=z-f(x,y)$, and the normal vector for each surface point is $\nabla F=\langle F_x,F_y,F_z\rangle=\langle f_x,f_y,-1\rangle$. $\mathbf{r}$ is an incident ray from one point of LCD screen to a surface point, $\mathbf{s}$ is the reflected ray from the surface point to the optical centre of CCD camera. $\mathbf{l}$ is the vector connecting the point at LCD screen to the CCD optical centre. $\mathbf{n}$ is the normal vector of the surface point. Here all of the vectors are expressed in terms of the CCD coordinate system. Hence, the relation between these vectors can be expressed as

$$\mathbf{r}=\mathbf{l}-\mathbf{s} \tag{1}$$

By conjugation of $\hat{\mathbf{s}}+\hat{\mathbf{r}}$, the normal can be expressed directly in terms of $\mathbf{s}$ and $\mathbf{l}$,

$$\hat{\mathbf{n}}=\hat{\mathbf{s}}-\hat{\mathbf{r}} \tag{2}$$

Where $\hat{\mathbf{s}}$, $\hat{\mathbf{r}}$ and $\hat{\mathbf{n}}$ represents the unit vector of $\mathbf{s}$, $\mathbf{n}$ and $\mathbf{r}$, respectively. Therefore, once the coordinate of the surface points are calculated, normal vector of these surface points can be calculated via Eq. (1) and Eq. (2).

2.2. Calculation of surface point and its normal vector

Assuming points $p_1(x_{m,n-1},y_{m,n-1},z_{m,n-1})$ and $p_3(x_{m,n+1},y_{m,n+1},z_{m,n+1})$ are adjacent to point $p_2(x_{m,n},y_{m,n},z_{m,n})$, and are each located at either side of p_2. Normal vector of point p_2 is $v_2\langle f_x,f_y,-1\rangle$.

Built on the fact that the normal vector is orthogonal to the surface curve at a given point, the relation among the three

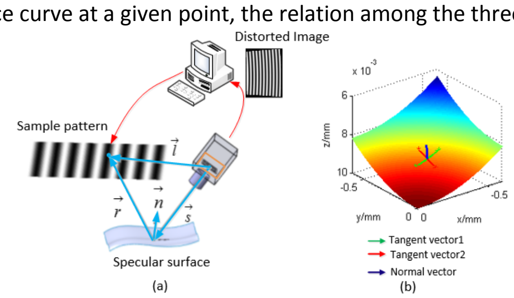

Figure 1. (a) Model of deflectometry, (b) tangent and normal vector.

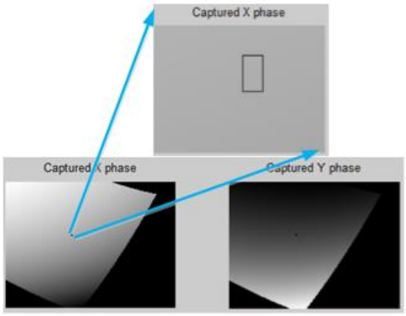

Figure 2. (Top) An area is selected acting as the overlapping region by stereo deflectometry, (down) simulated fringe phases in one CCD camera.

adjacent points can be approximately expressed as

$$f_x(x_{m,n+1}-x_{m,n-1})+f_y(y_{m,n+1}-y_{m,n-1})-(z_{m,n+1}-z_{m,n-1})=0 \quad (3)$$

Equation (3) means the normal vector of one surface point can be approximated to the perpendicular of the vector connecting the points at either side, as shown in Figure 1(b).

In the camera coordinate system, a surface point is locating at the ray intersecting the camera optical centre and one camera pixel, which means the surface point can be expressed as

$$(x_{m,n}, y_{n,n}, z_{m,n}) = k(s_x, s_y, 1) \quad (4)$$

Where $(s_x, s_y, 1)$ is the vector of the reflected ray.

On the basis of Eq. (3) and Eq. (4), if the surface points p_1, p_2 and their normal vector v_1, v_2 are known, the coordinate of surface point p_3 can be calculated. According to the description of section 2.1, normal vector v_3 of point p_3 can also be obtained by Eq. (1) and Eq. (2). Accordingly, in a practical measurement, if only the data on the overlapping region can be obtained, then the data on the non-overlapping region can be deduced from the values at the overlapping region with the proposed method.

3. Simulation work

To verify the proposed method, a convex surface $z=\sqrt{10000-x^2-y^2}$ with measurement range [-20, 20, -20, 20] mm is simulated to be under test using the deflectometry method. Additive noise with 0.03 rad standard deviation is added in the captured phase. Figure 2 shows the simulated fringe phase in the CCD plane. In this simulation, assuming the CCD internal parameters and system geometric relations have been calibrated, and a small region of the detected surface is selected acting as the overlapping field from stereo deflectometry, as shown in Figure 2(top). The physical coordinates and normal vector of the selected region are assumed to be known. On the basis of this knowledge, the information of the non-selected region of Figure 2 can be deduced using the proposed method. Figure 3 shows how the physical coordinate expands from the selected area to the whole surface. Figure 3(a) depicts that the selected region expands toward top using the proposed method. Then the calculated data in Figure 3(a) are regarded as the known points. After that, Figure 3(b) is computed on the basis of the data in Figure 3(a). Similarly, Figure 3(c) is calculated based on the data in Figure 3(b), and the physical coordinates of the whole surface shown in Figure 3(d) are calculated according to the result in Figure 3(c).

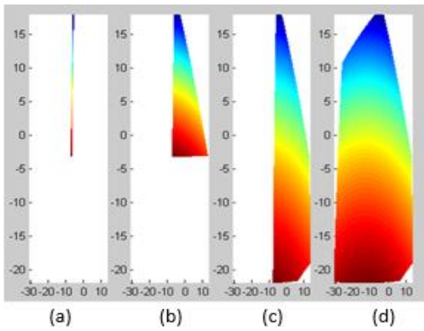

Figure 3. Physical coordinates are calculated from the selected region to the whole surface , (a) surface is expanded towards top from the selected region, (b) one-quarter surface is obtained based on (a), (c) is calculated standing on (b), (d) is computed based on the data in (c).

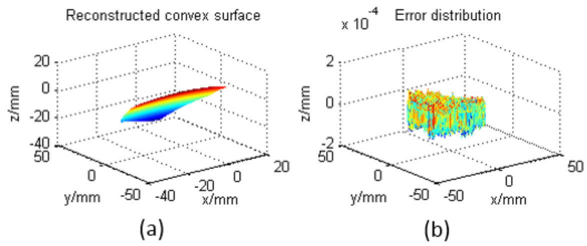

Figure 4. (a) Reconstructed surface, (b) errors distribution.

After the normal vectors of the whole surface are calculated based on the data from the selected region, which acts as the overlapping region in the stereo deflectometry, the whole surface shape can be obtained by integrating the normal fields with a 2D integration method [4-6], as shown in Figure 4(a). Figure 4(b) shows the error distribution between the reconstructed surface and the simulated surface shape. The standard deviation of the reconstruction errors is around 11.5 nm, which means the proposed method is feasible and effective to be used to calculate the data on the non-overlapping region.

4. Conclusion

A new method is proposed to estimate the normal vectors of non-overlapping region resulting from the data on the overlapping region in the deflectometry method, which is based on the approximation that the normal vector of one surface point is perpendicular to the vector connecting to the points at either side. Simulation on a convex surface verifies its feasibility and the result shows this method can be effective and accurate. Further work will be carried out on a practical measurement and error analysis.

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References

[1] Markus C. K., Jürgen K., and Gerd H. 2004 *Proc. SPIE* **5457**, 411-422.
[2] Marcus P. and Rainer T. 2005 *Proc. SPIE* **5869**.
[3] Balzer J. and Werling S. 2010 *Measurement* **43** (10) Elsevier 1305-1317.
[4] Guang H. L., Yang. Q. L., Ke L., Xu M. and Hai W. 2013 *J. Opt. Soc. Am. A* **30** 1448-1459.
[5] Lei H., Mourad I., Chao Z., Konstantine K. and Lin Z. *2015 Opt. Lasers Eng.* **64** 1-11
[6] Svenja E., Jürgen K. Markus C. K. and Gerd H. 2008 *Appl. Opt.* **47** 2091-2097.