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Design of optimized cross-spring pivots

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Abstract

A cross-spring pivot allows to achieve the rotation of a movable block via the deflection of leaf springs. When ultra-high precision is required, parasitic shifts have to be considered and the limits of applicability of approximated calculation algorithms have hence to be determined. The results obtained by employing these methods are thus compared with results obtained by using nonlinear finite element analysis (FEA), results obtained via the nonlinear Elastica approach as well as experimental data. FEA calculations allow also considering the influence of lateral loads and of non-symmetrical pivot configurations. An ultra-high precision design configuration allowing the minimisation of the parasitic shifts and of the variability of rotational stiffness, even for large rotations, is thus obtained.

Compliant mechanisms, rotation joints, cross-spring pivots, parasitic displacements, rotational stiffness, modelling

1. Introduction

Compliant mechanisms gain at least part of their mobility from the deflection of flexible members and are an alternative to sliding and rolling mechanisms used to transfer motion, energy and power. They are characterised by high precision, possibility of monolithic manufacturing, reduced costs and absence of backlash and wear. They are thus widely used in mechanical engineering design, precision engineering as well as the micro- and nanotechnologies [1]. A compliant rotational mechanism known as the cross-spring is characterised by high compliance along the 'in plane' rotational degree of freedom. When loaded with a pure couple M, it hence allows a movable block to rotate, via the deflection of leaf springs intersecting at their midpoints, with respect to the fixed block (Fig. 1). For larger rotation angles θ, however, the 'geometrical' centre of the pivot O moves to O', giving rise to a parasitic shift of amplitude d and phase φ that is detrimental to the precision of the analysed mechanisms [1-2].

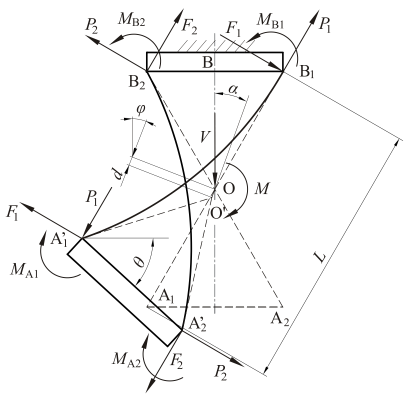

Figure 1. Cross-spring pivot

With the aim of determining the limits of applicability of the calculation methods proposed in literature to determine the parasitic shifts of cross-spring pivots, in this work are compared:

- results obtained via nonlinear finite element analyses (FEA),
- results obtained via the Elastica-type approach (EL) that takes into account the exact expression for the curvature of the springs in the geometrically nonlinear range [2-3];
- approaches where the approximated expression for the curvature of the beam (the square of the derivative is neglected in the curvature formula - AC) [1], or approaches based on the pseudo-rigid-body model (PRBM) [4-5], on kinematic (KM) [6] or on geometrical (GM) [7-8] considerations are used;
- experimental data on the behaviour of the cross-spring pivots reported in literature [1, 6, 9-12].

FEA calculations allow also considering alternative design configurations that allow minimising the parasitic shift and the variability of the rotational stiffness of the cross-spring pivot.

2. Finite element analysis

FEA calculations are performed by using the ANSYS® package, enabling nonlinear large deflection analyses of cross-spring pivots loaded with a couple M. The pivots are modelled via computationally efficient beam elements. With the goal of determining the respective limits of applicability depending on the needed accuracy, the results of nonlinear FEA (left scale on Fig. 2) are compared with the results of analytical calculations of different degrees of approximation. To enhance the visibility of the obtained differences, the right ordinate in Fig. 2 reports the differences $\Delta d/L$ of the dimensionless parasitic shifts obtained with the various methods with respect to FEA results. It can be observed that the EL results, comprising geometrical nonlinearities and taking into account the influence of axial loads on the bending of the beam, are in excellent agreement with FEA even for large rotations. Amongst the approximate analytical methods, the PRBM approach proposed in [4] gives rise to the smallest deviations. The AC, KM and GM approaches suggested, respectively, in [1], [6] and [8] are in good agreement with FEA for rotation angles $\theta \leq 15°$, whereas the PBRM results obtained by using the approach proposed in [5], as well as the GM results according to [7], allow only a first-degree approximation of the real behaviour of the pivot.

Among the results of experimental measurements reported in literature [1, 6, 9-12], only the recent interferometric measurements [1] are in good agreement with FEA results, since the difference between the two is smaller than 2 % even for the largest considered rotation angles.

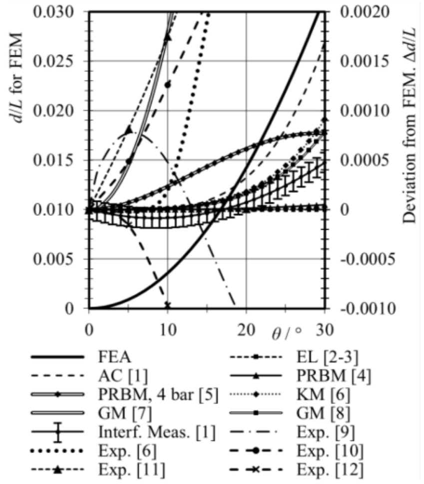

Figure 2. Results of calculations of parasitic shift amplitudes obtained with the considered methods

3. Influence of design parameters

The above analysis allowed verifying that FEA is suitable for modelling the behaviour of cross-spring pivots. It is thus used to analyse the effect of the variation of design parameters such as:

- angle α and position λ of intersection of leaf springs (Fig. 3a),
- initial curvature of spring-strips (Fig. 3b),
- monolithic configuration with the strips joined in point O, and
- external loads

on the entity of the parasitic shifts as well as the variability of rotational stiffness and the stresses occurring in the pivot.

Thorough FE analyses allow hence establishing:

- an increase of α causes a rise of the values of normalised parasitic shift amplitudes d/L and an exponential increase of the normalised rotational stiffness $KL/(EI)$ (with $K = M/\theta$);
- a change of parameter λ causes a substantial variation of the shifts and stiffness so that, for a symmetrical pivot ($\alpha = 45°$) with $\lambda \approx 0.13$, the parasitic shift is negligible even for large rotations θ, at the expense of a large increase of rotational stiffness and the stresses;
- an increase of springs' initial curvature induces an increase of stiffness, while a combination of spring-strips' inclinations $\gamma_1 = 15°$ & $\gamma_2 = 60°$ (Fig. 3b) results in the smallest parasitic shift;
- a monolithic configuration of the cross-spring pivot leads to a significant decrease of parasitic shifts (up to about 10 times) at the expense, however, of a marked increases of the stiffness (5 times) and the stresses (4 times) with respect to the conventional cross-spring pivot configuration of Fig. 1;
- a compressive vertical force V_C loading the pivot (along with M – see Fig. 1) narrows the stability range of the pivot, although inducing an increase of rotational stiffness and a decrease of parasitic shifts, whereas a tensile load V_T can induce a positive effect on pivot's stability with an increase of parasitic shifts;
- vertical loads cause in any case an increase of the stresses;
- when a tensile vertical force loads the pivot configuration with $\lambda = 0.1$ and $\alpha = 45°$, the variation of rotational stiffness is small in the whole range $V_T L^2/(EI) \leq 30$, while the corresponding parasitic shifts are still small (Fig. 4);
- finally, regardless of the orientation of vertical loads, a design configuration with $\lambda \approx 0.13$ and $\alpha = 45°$, for which the parasitic shifts are negligible, has an insignificant variation of rotational stiffness as long as $|VL^2/(EI)| \leq 10$ (zoomed region in Fig. 4).

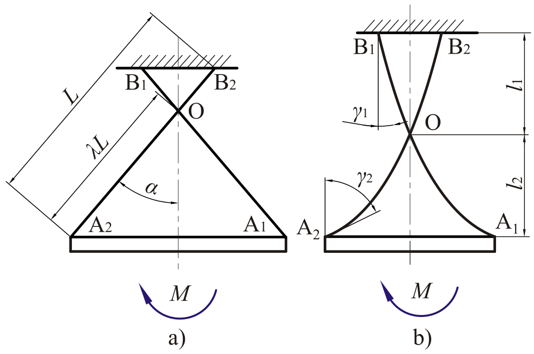

Figure 3. Non-symmetrical cross-spring pivot configurations

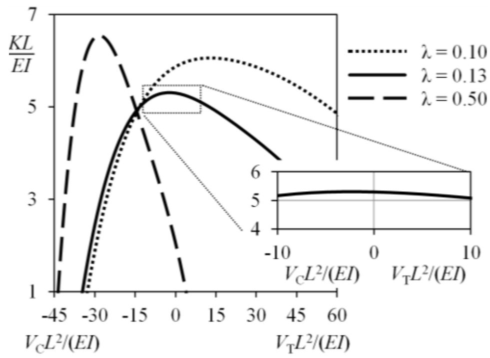

Figure 4. Rotational stiffness depending on vertical loads for a symmetrical cross-spring pivot with $\alpha = 45°$

4. Conclusions and outlook

A design configuration of the cross-spring pivot with $\lambda \approx 0.13$ and $\alpha = 45°$ allows ultra-high precisions to be achieved since it is characterised by negligible parasitic shifts and rotational stiffness variations even for large pivot rotations. This is accomplished, however, at the expense of an increase of the value of pivots' stiffness. In future work, a design configuration of a compensated cross-spring pivot, allowing larger rotations, will be considered, still with the aim of maintaining the parasitic shifts and the variability of the rotational stiffness at negligible levels.

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