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A novel approach towards vibration suppression in linear direct feed drive

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Abstract
Linear direct feed drives are widely used in high performance equipment and machine tools. However, the abrupt counter force to the machine frame from the secondary part of the linear motor induces possible residue vibration. The jerk decoupling cartridge (JDC) provides a buffer to reduce such an impact, but no existing JDC design considers the factors of closed-loop control together with mechanical parameters, as well as how to obtain optimal parameters under various mechanical and electrical design constraints. This paper presents a linear quadratic optimization approach to design the controller and the JDC. The problem is correctly formulated and the correctness of obtained optimal parameters is verified through a series of simulation.

Keywords: linear direct feed drive, vibration, jerk decoupling, optimal parameters.

1. Introduction

Linear direct feed drive is an excellent choice to meet the requirement of high speed, ultra-high accuracy and improved reliability due to the simplicity of its mechanical structure. One major disadvantage is the jerk dependent reaction force on the secondary part of the linear motor [1]. In this way, the abrupt counter force to the machine frame from the secondary part induces residue vibration. Jerk decoupling technique is a popular approach to decouple the reaction force from the machine frame, where a spring-damper system is implemented between the secondary part of the linear motor and the machine frame as a buffer to reduce residue vibration [2]. In this extended abstract, the jerk decoupling technique is applied and a constrained linear quadratic optimization method is used to design the controller and the JDC concurrently.

2. System modelling

A mechanical linear direct drive mounted on a machine structure through JDC is shown in Figure 1. In this model, masses of the primary and the secondary part of the linear motor are represented by m_1 and m_2. Spring constant, damper coefficient, force constant and back EMF constant are represented by k_2, b_2, K_f, b_{12} respectively. Jerk decoupling force applied to the machine frame is represented by f. Current and terminal voltage of the linear motor are represented by i, and u_m. Notations y_i, $\dot{y}_i$ $\ddot{y}_i$ represent the displacement, the velocity and the acceleration of m_i, where $i = 1, 2$.

By ignoring the bearing friction, equations of motion can be expressed as follows:

$$F = m_1\ddot{y}_1$$

$$F = K_f\, cos(\omega y_2) i(t)$$

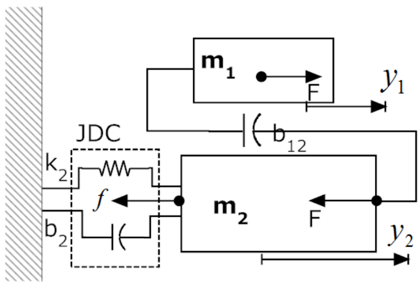

Figure 1. Modelling of feed drive mounted on a machine structure

$$-F - f = m_2\ddot{y}_2$$

$$f = k_2 y_2 + b_2\dot{y}_2$$

$$u_m(t) = b_{12}(\dot{y}_1 - \dot{y}_2) + Ri(t) + Ldi/dt$$

It is assumed that electrical time constant is much smaller than mechanical time constant and the motion of m_2 is small enough compared to its magnetic pole pair length. In other words, $cos(\omega y_2)$ and Ldi/dt are ignored.

The objective is to design the PID feedback controller, feed-forward controller and JDC concurrently to minimize the vibration, meanwhile taking care of other design factors. To seek the optimal trade-off among primary part tracking error $\widetilde{y_1}$, secondary part vibration y_2, terminal voltage u_m and jerk decoupling force f, the functional cost J is to be optimized, where

$$J = \int_0^T \left(q_{11}\widetilde{y_1}^2 + q_{22}{y_2}^2 + r_{11}{u_m}^2 + r_{22}\dot{u_m}^2 + r_{33}f^2 + r_{44}\dot{f}^2\right)dt,$$

In the cost function, $\dot{u_m}$ and $\dot{f}$ are the derivatives of u_m and f respectively, they are used as augmented inputs for linear quadratic problem formulation and optimization stage only.

3. Problem formulation

Define the state vector:

$$x^T = \left[\int_0^t y_1 d\tau \quad y_1 \quad \dot{y_1} \quad \ddot{y_1} \quad y_2 \quad \dot{y_2} \quad \ddot{y_2}\right] = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7]$$

The state space is given by:

$$\dot{x} = Ax + Bu,\ y = Cx$$

Define $\overline{C} = I - C^T(CC^T)^{-1}C$, then $\overline{y} = \overline{C}x$, where $\overline{y}$ is the part of the state vector that is not seen by $y = Cx$. The cost function is re-formulated by adding in quadratic term of $\overline{y}$ into the original cost function, which aims to convert output feedback to state feedback. The augmented system is represented by:

$$\dot{x_a} = A_a x_a + B_a u$$

The cost function of the augmented system is re-written as:

$$J_1 = \int_0^T (x_a^T Q_a x_a + u^T R u) dt$$

The decentralized composite controller is given by:

$$u = k_a x_a,$$

where

$$k_a = \begin{bmatrix} -k_i & -k_p & -k_d & 0 & 0 & 0 & 0 & k_{z1} & k_{z2} & k_{z3} & k_{z4} & 0 \\ 0 & -k_i & -k_p & -k_d & 0 & 0 & 0 & 0 & k_{z1} & k_{z2} & k_{z3} & k_{z4} \\ 0 & 0 & 0 & 0 & k_2 & b_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_2 & b_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

For the primary part, a PID controller is implemented together with a feed-forward controller. For the secondary part, JDC is used for regulating purpose and it is regarded as a mechanical PD controller. The block diagram of the control system is shown in Figure 2.

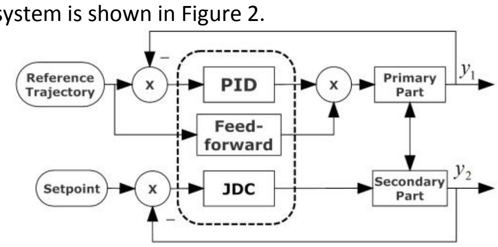

Figure 2. Schematic of JDC with 2-DOF control

Under the assumption of randomly uniform distribution of initial state errors $x_a(0) - x_d(0)$ within a bound, the cost function J_1 is equivalent to:

$$J_2(K_a) = \text{trace}\left\{\int_0^T [e^{(A_a+B_a k_a)t}]^T (Q_a + {k_a}^T R k_a) e^{(A_a+B_a k_a)t} dt\right\},$$

The optimization problem is to minimize the cost $J_2(K_a)$ subjected to two constraints: $k_{z1} = 0$ (no position feedforward term for the system with at least one inherent integrator) and $A_a + B_a k_a < 0$ (keep the closed-loop stability of the system).

4. Simulation

Standard linear quadratic optimization is based on full state feedback, but in the actual feed drive, both of the force feedback from mechanical joints and the controller feedback are only based on part of states, so there is no standard closed-form solution to the optimization problem. To solve the nonlinear optimization problem, fmincon solver and interior-point algorithm are used for optimization in MATLAB. Weighting matrices Q and R are chosen based on Bryson's Rule [3]:

$$Q = \text{diag}\left(\frac{1}{{z_1}^2}, \frac{1}{{z_2}^2}\right), R = \text{diag}\left(\frac{1}{{u_1}^2}, \frac{1}{{u_2}^2}, \frac{1}{{u_3}^2}, \frac{1}{{u_4}^2}\right),$$

where $z_1$and z_2 are maximum acceptable values of $\widetilde{y_1}$ and y_2, u_1, u_2, u_3 and u_4 are maximum acceptable values of $u_m, \dot{u_m}, f$ and $\dot{f}$ respectively. z_1, z_2, u_1 and u_3 is specified according to design specifications, where $z_1 = 0.03$, $z_2 = 0.03, u_1 = 200, u_3 = 500$. As $\dot{u_m}$ and $\dot{f}$ are not considered in design specifications, u_2 and u_4 can be set large to make their weightings small. In the simulation, weighting matrices are chosen as: $Q = \text{diag}\{3\,000, 1\,000\}, R = \text{diag}\{2.5 \times 10^{-5}, 6 \times 10^{-12}, 4 \times 10^{-6}, 6 \times 10^{-12}\}$. They are regarded as the starting point of a trial-and-error iterative design procedure to obtain desirable design specifications.

It follows that optimal composite gain matrix K_a is given by:

$$\begin{bmatrix} 0 & -569 & -109 & 0 & 0 & 0 & 0 & 0 & 1.7 & -1.3 & -0.4 & 0 \\ 0 & 0 & -569 & -109 & 0 & 0 & 0 & 0 & 0 & 1.7 & -1.3 & -0.4 \\ 0 & 0 & 0 & 0 & 968 & 262 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 968 & 262 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

which achieves an optimal functional cost $J_2(K_a) = 7.0 \times 10^3$.

Consider the AUM4-S4 parallel type linear motor, simulation results of primary part tracking error, secondary part vibration, terminal voltage, derivative of terminal voltage, jerk decoupling force and derivatives of jerk decoupling force are shown in Figure 3 respectively. It shows that optimal gain matrix gives good performance in tracking accuracy and vibration.

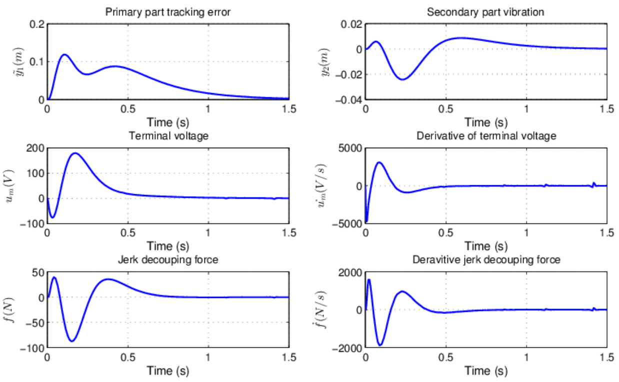

Figure 3. Simulation results of proposed control system

5. Conclusion

This paper presented a systematic methodology to design the controller and the JDC for high-speed feed drive positioning. The cost function was initiated with consideration of various design factors. The derivative of the terminal voltage and the derivative of the jerk decoupling force were augmented as two inputs for linear quadratic problem formulation and optimization stage. The constrained linear quadratic optimization problem was correctly formulated, controller parameters and JDC parameters were optimized. Simulation results showed that obtained optimal parameters gave good tracking performance and small vibration of the linear motor.

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