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Design against fracture of piezoelectric layers used within smart systems

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Abstract

The wide use of piezoelectric materials in many smart systems motivates the current need of predicting very effectively their reliability. A key issue of design of the piezoceramic layers is preventing some damage like fracture, fatigue and creep. Prediction of crack propagation through a piezoelectric layer while its active functions are exploited is rather difficult. Analytical methods already proposed in the literature often fail when either geometry is more complicated than a rectangular patch or structural behaviour becomes nonlinear because of a large vibration amplitude, typical, for instance, of the vibration energy harvesters. A research activity was therefore aimed at assessing a reliable numerical tool to combine the prediction of crack propagation within the piezoelectric layer through the Finite Element Method and the analysis of the electromechanical coupling between strain and voltage. The crack behaviour inside the smart material is described by calculating the so-called Stress Intensity Factor (SIF), the J-integral and the crack path, but in this case local effects induced by the piezoelectric phenomenon are included. The ABAQUS code was used to investigate both the fracture mechanics and the piezoelectric phenomenon, by means of the ISIGHT tool which connected the two environments of the code which perform separately those analyses. This approach allowed detecting the reaction applied by the electromechanical coupling to the crack propagation at its tip, in operating conditions. The above mentioned tools and results allowed investigating some particular effects of piezoelectric phenomenon on the smart structure behaviour in service.

Keywords: Vibration energy harvesting, Fracture mechanics, Piezoelectric materials, Finite Element Method, Structural Mechatronics

1. Introduction

Prediction of damage in composite structures equipped with functional materials like piezoceramics is currently a key issue of design in several applications of mechatronics like active vibration control, structural health monitoring and vibration energy harvesting. To effectively predict the structure life time the crack propagation has to be investigated in terms of fracture mode (I, II, III or even mixed), crack path and propagation rate [1]. This task is rather difficult in mechanical components, when geometry is more irregular than a circular or rectangular plate. Moreover, behaviour of smart layers might be nonlinear in case of large vibration amplitude as in flexible energy harvesters. In addition it is worthy noticing that in piezoelectric material state of stress and of strain depend not only on the mechanical behaviour but even on the electromechanical coupling thus changing the local distribution of displacements around the crack tip [2]. As it looks intuitive when the layer is active, i.e. strain is induced either by the electric field or by the bonded structure, the local stiffness of material changes, thus affecting the reaction of material to the crack propagation. Sometimes electromechanical coupling contributes to improve the mechanical strength in spite of analytical predictions [3].

In the literature some analytical tools are available to predict the fracture mechanics of solids. The so-called 'Stress Intensity Factor' (SIF) is used to analyse the coupled effect of crack length and stress induced at its tip. Energy released rate of crack is usually computed by resorting to the J-integral (JI). It describes the energy of a region around the crack tip limited by a selected contour as C_1 and C_2 in Fig.1. This integral is applied to the total length of contour *C*, is defined by a local coordinate *s* and by a orthogonal versor *n*, and includes the density of elastic energy associated to strain, W_e, the stress applied to the contour, T, and the corresponding mechanical displacement *u*.

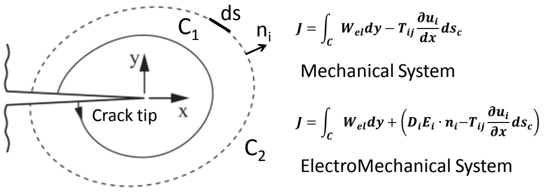

Figure 1. Definition of J-integral applied to fracture mechanics of a mechanical system and to piezoceramic material.

Crack propagation can be investigated by resorting to the well known relation between J-integral and the energy release rate G [1]:

$$\boldsymbol{J} = G = -\frac{\partial W}{\partial a} \tag{1}$$

where W describes the total energy associated to the crack system and *a* is the crack length. Some authors (as [4]) investigated the role of piezoelectric effect upon the crack propagation through analytical approaches, by including into the J-integral the electromechanical energy associated to the electric field (Fig.1). Electromechanical coupling in piezoelectric material is described by the coupled constitutive laws of material as follows:

$$T_{ij} = C_{ijkl}(S_{kl} - S^r_{kl}) - e_{kij}E_k \tag{2}$$

$$D_i = e_{ijk}(S_{jk} - S^r_{jk}) + \varepsilon_{ij}E_j + P_i \tag{3}$$

Symbols are mechanical stress components T_{ij} and strains S_{kl}, being S^r_{kl} the residual strain induced by polarization P_i; C_{ijkl} are elastic coefficients, being e_{kij} the piezoelectric coefficients, E_k the electric field and D_i the electric displacement along a given direction *i*, while ε_{ij} is the dielectric permittivity of material [3]. The main problem in case of industrial applications is that geometry sometimes does not allow an easy prediction of coupled behaviour in presence of crack. Moreover, in many commercial Finite Element codes the two analyses, dealing with fracture mechanics and piezoelectric effect, are usually run separately in different tasks of numerical solution.

2. Proposed numerical tool and results

The ABAQUS code is unable to analyze simultaneously the fracture behavior of a piezoelectric material and its electromechanical coupling. Nevertheless, these two analyses can be performed separately. To overcome this problem, a new procedure was built up and tested, by connecting the two above mentioned solution tasks (Fig.2). Structure is first represented inside the ABAQUS code, by providing all the mechanical properties of material and relevant information about crack (load, boundary conditions, geometry, initial length), then it is meshed. To input the electrical properties of the piezoelectric layer the same model is developed as a separated case, through piezoelectric elements. Prediction of fracture in presence of piezoelectric phenomenon is performed by running the two analyses as a sequence, thus making the two above mentioned models interacting each other, inside the ISIGHT tool. As Fig.2 shows the tool developed computes the stress intensity factor (SIF) and the J-integral (JI). When piezoelectric layer just behaves like a sensor, i.e. no external voltage is applied, model of the cracked structure described into the ABAQUS code is transferred into the ISIGHT tool in subroutine 'Crack' which computes SIF and JI, as in case of a simple metal.

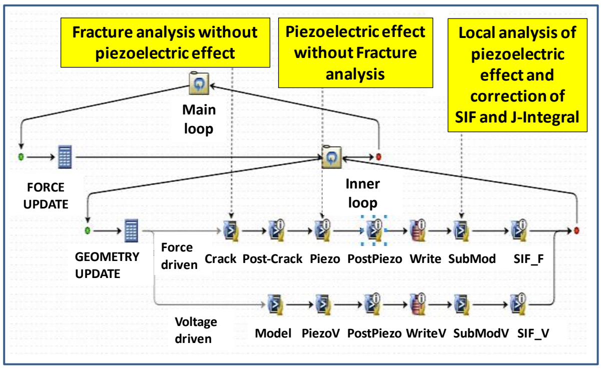

Figure 2. Toolbox developed to perform the prediction of crack propagation with piezoelectric effect.

Results are collected by 'Post-Crack' and used in 'Piezo' together with the second model including the piezoelectric elements to compute displacements induced by the mechanical actions and the voltage distribution. The new configuration of voltage induces a local piezoelectric effect which affects the boundary conditions of load around the crack tip. They are written ('Write') and used ('SubMod') to refine both the SIF and JI previously calculated, which now include the electromechanical coupling. Then they are shown by subroutine 'SIF_F'. This is done for each load step through an iterative solution and convergence is assured step by step. When strain is induced by the applied electric field, i.e. piezoelectric layer behaves as an actuator, option 'voltage driven' is followed. Task 'Model' computes the loads applied to the structure as a consequence of the electric field excitation, and both the preliminary SIF and JI are found as in option 'Force driven'. Then correction due to the capability of sensing still present in the layer is calculated by 'SubModV' and corrected SIF and JI are shown through the 'SIF_V' subroutine.

This toolbox was validated on a case proposed in literature, then used to analyze the behaviour of piezoelectric composite structures. In a single mode of fracture of piezoelectric material mechanically loaded crack might induce the rupture of material, while its propagation stops at a certain length when voltage acts as a driving load. To revamp propagation a larger electric field is required. This 'barrier effect' is due to boundary conditions imposed by the piezoelectric effect at crack tip. In case of a mixed mode (Fig.3), if mechanical load is applied, crack changes its direction, but deviation is smaller when load is driven by voltage. In composite structures, with metallic substrate and surface bounded piezoelectric, crack propagates along a straight line under both mechanical force and voltage excitation in single mode. In mixed mode propagation stops when crack tip reaches the interface between materials, i.e. under mechanical action propagation changes direction at the interface and breaks both materials, while in actuation crack never breaks the piezoelectric layer, being stiffened by voltage.

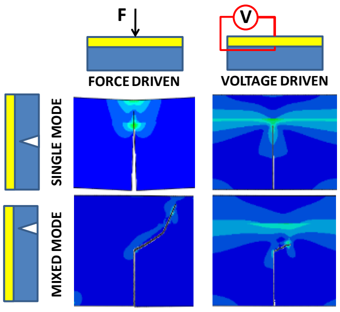

Figure 3. Crack propagation in composite structure with upper bonded piezoelectric layers with middle and lateral crack, under mechanical or electromechanical loading.

3. Conclusion and design criteria

This study demonstrates that a beneficial stiffening of smart layer in operation makes slower the crack propagation, when material is controlled by voltage, thus motivating some observed higher strength in service. This effect could be used to reduce the thickness of smart layers, thus making the system lighter, stronger in coupling (it is inversely proportional to thickness) and less expensive. Nevertheless, a suitable compromise among the needs of having a sufficient cross section for strength, thinner smart layer, low voltage actuation has to be still found through a further research activity.

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