

## Uncertainty of measurement for the inertial torque estimation in angular acceleration ramps of rotating shafts

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### Abstract

The control of the inertial reactions of rotating shafts is more commonly made through measuring the angular acceleration with encoders than using inline torque sensors. A feasible methodology for the characterization of this proceeding to estimate the inertial torques in specific torque load regimes is presented. Results show that the parameters of this estimative will depend on the data processing method, on the region of the torque load curve analysed and also on the angular speed intervals of the acceleration ramps.

Keywords: inertial torque, rotary machines, data processing, uncertainty of measurement

### 1. Introduction

In rotating machines or assemblies, the inertial torque caused by the angular acceleration of the shaft's mass moments of inertia is normally an undesired component and works as a restriction to the process. Therefore, angular acceleration regimes are often over dimensioned, with acceleration periods set to last longer and curves to be smoother [1, 2].

To better control this inertial reactions in real time, the measurement of angular speed with encoders is more common than the use of inline torque sensors. With a more accurate estimation of the inertial torque response, the driving of the shaft can reach better performance with faster accelerations in different speed steps.

When measuring time-varying mechanical quantities, such as angular speed, the dynamic error is introduced, namely the ratio between the input and the response cannot maintain itself, as in static measurements. In order to mitigate these errors, the measured signal needs to be post-processed. This paper presents a methodology to evaluate the contribution of processing methods to the final estimated value of inertial torque in critical parts of the load curve.

### 2. Methodology

The methodology is based on an experiment carried out with a rotating shaft with simultaneous measurement of speed and torque followed by data processing. Applying acceleration to the shaft with profiles similar to load impulses, the non stabilization levels and the higher torque rates (Fig. 1, regions II and III) are characteristics that simulate the severe dynamic nature of some industrial applications, compared to the expected torque peak curve (region I).

According to the theory, the relationship between torque and angular acceleration magnitudes should follow the physical principle of Newton's second law applied to angular movements, as shown in Eq. (1):

$$\tau = \theta \cdot \dot{\omega} \therefore \theta = \frac{\tau}{\dot{\omega}} \quad (\text{Eq. 1})$$

where;

$\theta$  moment of inertia of the shaft ( $\text{kg}\cdot\text{m}^2$ )

$\omega$  angular speed ( $\text{rad}\cdot\text{s}^{-1}$ )

$\dot{\omega}$  angular acceleration ( $\text{rad}\cdot\text{s}^{-2}$ )

$\tau$  generated inertial torque ( $\text{N}\cdot\text{m}$ )

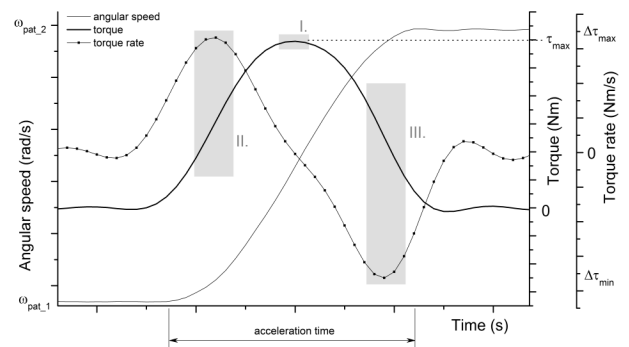


Figure 1. Speed ramp from  $\omega_{\text{pat}_1}$  to  $\omega_{\text{pat}_2}$ , torque curve and torque rate profile.

Thus, these torque regions can be correlated to the same regions in the acceleration curves. The measurements were carried out in two speed intervals from  $157 \text{ rad}\cdot\text{s}^{-1}$  to  $209 \text{ rad}\cdot\text{s}^{-1}$  and from  $209 \text{ rad}\cdot\text{s}^{-1}$  to  $261 \text{ rad}\cdot\text{s}^{-1}$ . The acceleration time was set to 2 s in the variable speed drive and the driving curves were repeated three times. An acquisition rate of 50 Hz was used. The acceleration data is obtained by differentiation of speed on time, following by the data processing. Four common processing sequential methods were chosen to be evaluated for their influence on the final estimation of  $\tau_i$ :

*Method A:* filtered speed signal -> differentiation.

*Method B:* raw speed signal -> differentiation -> smoothing of acceleration data.

*Method C:* smoothing of the speed signal -> differentiation.

*Method D:* raw speed signal -> differentiation.

The rate between the measured torques and the calculated accelerations corresponds to calculated values of mass moment of inertia ( $\theta_{calc}$ ). The deviation amplitude and variation of these values of mass moment of inertia can be considered to evaluate the method contribution and also as a source of uncertainty.

### 3. Results

Results are divided in two main parts. The deviations of the calculated mass moments of inertia are presented first and then the identification of their impacts in a broader uncertainty approach for the uncertainty of the estimated inertial torque.

#### 3.1. Calculated mass moments of inertia

Table 1 shows the weighted means and standard deviations for the values of  $\theta_{calc}$  on each method in the peak region of the curve (region I in Fig 1).

**Table 1** Results of mean and standard deviation of  $\theta_{calc}$  for 'Region I' ( $\text{kg}\cdot\text{m}^2$ ) in the different speed intervals.

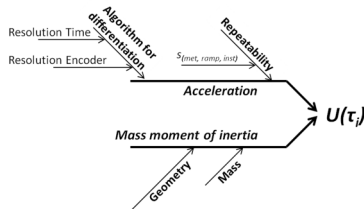
Methods	A	B	C	D
<i>Speed interval: 157 rad s<sup>-1</sup> to 209 rad s<sup>-1</sup></i>				
Means / $\text{kg}\cdot\text{m}^2$	$7.41 \cdot 10^{-3}$	$7.40 \cdot 10^{-3}$	$7.41 \cdot 10^{-3}$	$7.41 \cdot 10^{-3}$
Weighted StDev	1.16%	2.77%	2.94%	5.03%
<i>Speed interval: 209 rad s<sup>-1</sup> to 261 rad s<sup>-1</sup></i>				
Means / $\text{kg}\cdot\text{m}^2$	$7.44 \cdot 10^{-3}$	$7.41 \cdot 10^{-3}$	$7.41 \cdot 10^{-3}$	$7.42 \cdot 10^{-3}$
Weighted StDev	0.60%	0.64%	0.64%	0.86%

In the first speed interval,  $\theta_{calc}$  values were more unstable and the application of processing methods are more efficient. In the second speed interval, results were more stable and the processing methods showed compatible results.

According to [3] the measurable mass moment of inertia of the transducer is  $7.92 \cdot 10^{-3} \text{ kg}\cdot\text{m}^2$ , which gives a systematic difference of about 6% to the values of  $\theta_{calc}$ . This can be a result of using the static calibration of the torque transducers as a reference for this dynamic evaluation.

#### 3.2. Uncertainty of Measurement

The sources for the evaluation of the uncertainty of measurement of the inertial torque are identified in figure 2.

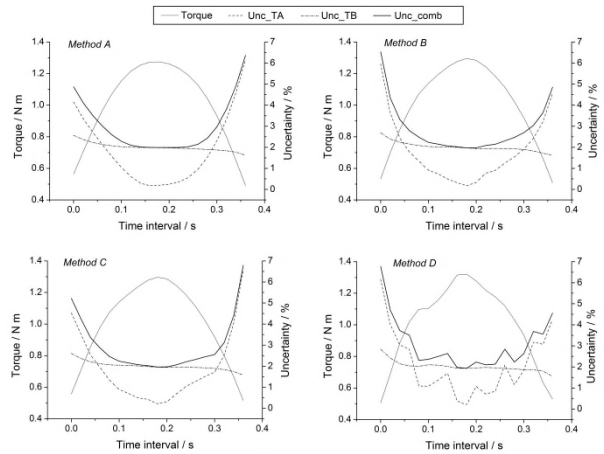


**Figure 2.** Sources for the uncertainty of measurement of  $\tau_i$ .

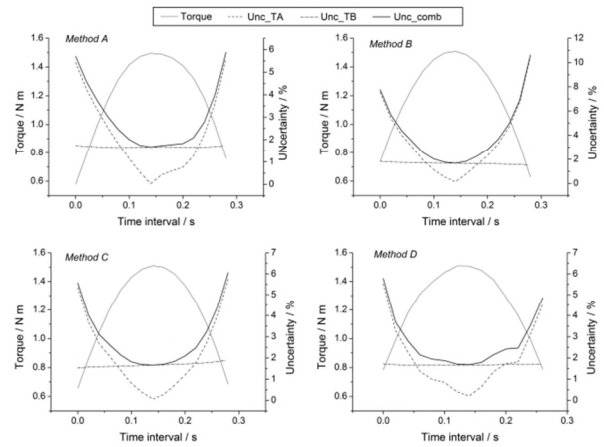
The acceleration sources have the algorithm as a type B and the repeatability as a type A, which the standard deviation is indexed according to the method, ramp and instant of time. The differentiation algorithm uses a difference formula to approximate the derivative, it means, for an acceleration value, two angular speed values are used.

The acquisition system has a resolution of time about  $1 \cdot 10^{-9} \text{ s}$  and this will be used as its uncertainty source in the same way as the resolution of  $1^\circ$  for the encoder. The mass moment of inertia used is the values showed in [3] with an estimated uncertainty of 1%.

Figures 3 and 4 show the mean torque curves and the profile of uncertainties (type A, type B and combined) on each method for the speed intervals from  $157 \text{ rad}\cdot\text{s}^{-1}$  to  $209 \text{ rad}\cdot\text{s}^{-1}$  and from  $209 \text{ rad}\cdot\text{s}^{-1}$  to  $261 \text{ rad}\cdot\text{s}^{-1}$  respectively.



**Figure 3.** Estimated inertial torque curves and uncertainties profiles for each method in the speed interval of  $157 \text{ rad}\cdot\text{s}^{-1}$  to  $209 \text{ rad}\cdot\text{s}^{-1}$ .



**Figure 4.** Estimated inertial torque curves and uncertainties profiles for each method in the speed interval of  $209 \text{ rad}\cdot\text{s}^{-1}$  to  $261 \text{ rad}\cdot\text{s}^{-1}$ .

It is clear that there is a predominance of type A in the borders of the torque curves, i.e. regions II and III. Differently, around region I, with more stable data, the uncertainty for this peak area has a predominance of type B sources, resulting in smaller coverage factors ( $k$ ). As an example, for the probability of 95%,  $k$  can reach the value of 2.0 in the peak region while reaching a value of 4.0 (4.3 in the worst case of method D) in the borders.

### 4. Conclusion

In this paper a methodology of evaluating the estimation of inertial torques in rotating shafts is presented. Results showed that the more unstable is the signal, more influent will be the post-processing method. The analysis also demonstrates that regions II and III have much larger uncertainties than region I. Next works should focus on repeating the methodology with different parameters and better reference equipment.

### References

- [1] Claudino R, Gusmão L, Oliveira R S, Kalid R, Lepikson H, Fröhlich T, *Estimativa do torque inercial em eixos girantes - uma abordagem metrológica ao processamento de sinais*, III CIMMEC, SBM, 2014.
- [2] Oliveira R S, Lepikson H, Winter S, Theska R, Fröhlich T, Bitencourt A C, Machado R, *New proposals for the dynamic tests of torque transducers*, 58<sup>th</sup> IWK, TU-Ilmenau, 2014.
- [3] HBM-GmbH, *T10F - Torque transducer - Data Sheet*.