# Determination of volumetric errors using a machine tool model employing a reduced set of geometric errors

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## Abstract

The presented article describes the preparation of a mathematical model for calculating volumetric errors using homogeneous transformation matrices. Position-dependent and independent geometric errors are included and the method of their inclusion in the calculation model is presented. Using the method described, the computational model is designed for a specific configuration of a three-axis and five-axis machine tool. Using sensitivity analysis, a reduced set of geometrical errors is selected, having a major impact on the working accuracy of the specific machine configuration. Based on the measured geometric errors and the derived mathematical model, simulations are performed, verifying the functionality of the entire process.

## 1. Introduction

The volumetric error is the deviation of the final position of the tool relative to the workpiece. It represents the summarization of all geometric errors that are present on a specific machine tool. Schwenke *et al.* [1] present two procedures for obtaining values of volumetric errors: a) Direct measurement of the volumetric error at specific points of the workspace b) Calculation of volumetric errors from individually measured geometric errors. The homogeneous transformation matrices (HTM) are the most commonly used to calculate the volumetric error. The principles of this procedure for three-axis milling machines were provided by Schultschik [2] and Wang [3]. In all 21 geometric errors are considered for three-axis machine tools. The situation is more complicated in the area of five-axis machine tools, which combine the linear and the rotational axis. Hong *et al.* [4] derived a kinematic model of a tilting rotary table including movement and position error. A complex model of a five-axis machine tool was derived by Bohez *et al.* [5]. This model considers 37 position-dependent and independent geometric errors. Since the number of geometric errors considered in this model is relatively high, many authors have chosen the method of reducing the total number of considered parameters. Udin *et al.* [6] designed a model of a five-axis machine tool, which operates with a set of 11 kinematic errors. Brecher *et al.* [7] used a sensitivity analysis to reduce the total number of geometric errors considered in a model. The model of a five-axis machine tool operates with a set of 42 geometric errors. The total number of errors considered was reduced to 21 errors using a sensitivity analysis.

## 2. Development of a machine´s volumetric error model

The machine tool is considered as a system of rigid bodies, which are interconnected by kinematic constraints representing the machine’s moving axes The HTM are used for kinematic chain description including necessary dimensions of the machine structure, movement errors and location errors of each axis. There are 6 errors of movement for each machine axis according to 6 degrees of freedom of a solid body moving in space. Location errors of one axis relative to the other axis are represented by squareness errors. The volumetric error vector is obtained by subtracting the ideal machine kinematic chain (without geometric errors) from the real one (with geometric errors included). Based on the above-mentioned principles, the volumetric models for the selected three-axis (FXYZ configuration) machine and five-axis (CAFYXZ configuration) machine are formulated.

Geometric errors of movement and location for each axis are measured on a real machine and used as input data for model simulations.

### 2.1 Introduction of geometric errors to volumetric model

The volumetric error model contains position-dependent and independent geometric errors. The geometric errors of the axis movement are position dependent because their values change with the axis coordinate. Therefore, they are expressed as functions of an appropriate machine axis coordinate. Linear interpolation is used for expressing the function. It is the most accurate way to obtain the values from the measured data.

### 2.2 Model simplification and the reduction of the number of errors

The volumetric model obtained from kinematic chains contains a large number of summands. The following assumption is introduced to simplify the calculation: Angles of angular errors are very small, therefore $\sin\alpha = \alpha$ and $\cos\alpha = 1$ can be considered. The multiple of two errors can be ignored as a higher order error.

In order to reduce the total number of geometric errors considered, a sensitivity analysis was performed. The analysis consisted of the calculation of volumetric error vector magnitude, while in each calculation one of the geometric errors was neglected. The result was compared with the calculation considering all geometrical errors. The influence of a geometric error on the volumetric error is considered as negligible if the difference is less than ± 0.5 %. The number of the considered geometric errors was reduced from 21 to 16 for the three-axis machine (FXYZ configuration) and from 37 to 25 for the five-axis machine (CAFYXZ configuration).

## 3. Model verification

Verification of the model was performed by comparing the results of the calculation and the measurement of circular interpolation. An example of the results comparison for the three-axis machine is shown in Figure 1.

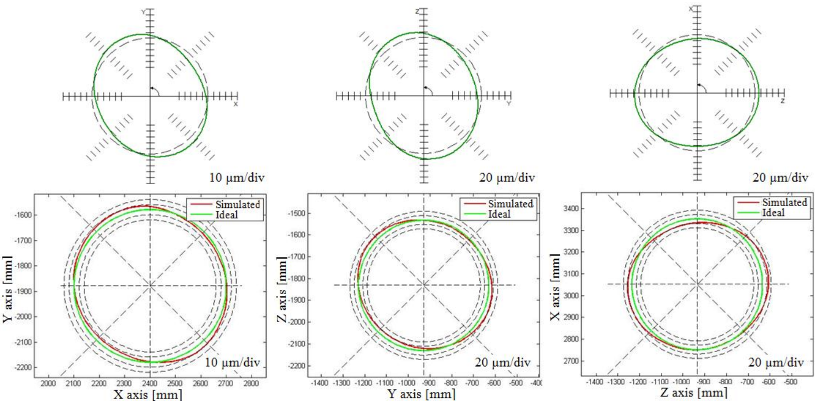


Figure 1: Comparison of computed and measured circular interpolation

Individual geometrical errors were measured on the machine using standard methods and used as input data for the mathematical model. The model was used for simulation of the circular interpolation in three coordinate planes and different positions. The results obtained were compared with the measurement using the QC10 Ballbar. In all cases, the radius of the circle tested was 300 mm and feed rate was 500mm/min.

## 4. Summary and conclusion

Achieving sufficient agreement between the simulation and measurement shows that the model can be used for further research in the area of increasing the accuracy of machine tools. A useful tool was also obtained to determine the significance of geometric errors for a specific machine configuration. This allows reducing the time requirements needed to measure and calibrate the machine and making the whole process more efficient.

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