

# **Nanometer accurate orbit model for analysing the error motion of a porous aerostatic bearing**

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## **Abstract**

In this work, a nanometer accurate 2D-orbit model is developed for analysing the influence of the minutest details affecting the radial error motion of a porous aerostatic journal bearing. This allows us to increase the running accuracy of an axis of rotation system, such as an aerostatic spindle or rotary table, to the sub-nanometer level. The orbit model is validated experimentally by measuring the radial error motion of a porous aerostatic rotary table with the use of a special measurement technique. The difference between the outcome of the orbit model and radial error motion of the rotary table under test is only 6 nm.

## **1 Introduction**

The axis of rotation error motion (ANSI/ASME B89.3.4M standard), designated as error motion in this work, of a well-designed air bearing system will be mainly determined by the machining accuracy of the bearing surfaces. This is so because the clearances of air bearings should be made as small as possible in order to obtain a high stiffness. Hence, the radial error motion of an aerostatic journal bearing with inherent feedholes can be reduced most effectively by increasing the number of feedholes  $N_f$ , as shown in [1,2]. The radial error motion of an aerostatic porous journal bearing is analysed in this work as this ideally has an infinite number of feedholes and consequently a very low error motion.

## **2 Mathematical model of a porous journal bearing**

An externally pressurized porous gas journal bearing is shown in Fig. 1. Pressurized air at constant pressure  $p_s$  is fed through the porous material wherein the pressure drops to  $p'$  and then to  $p$  in the bearing clearance  $c$ . Finally, the air exhausts to the

atmosphere at a pressure  $p_a$ . The ends of the porous material are sealed in order to avoid end leakage.

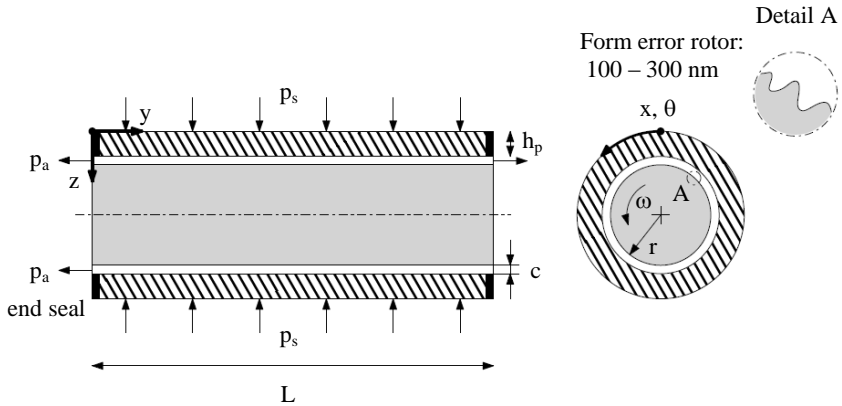


Figure 1: An aerostatic porous journal bearing.

## 2.1 Orbit model

It is customary to assume that the bearing surfaces are geometrically perfect in air bearing analyses. This assumption is only valid if the clearances are very large compared with the form errors of the bearing surfaces. In order to obtain the orbit data of the rotor centre, the form error of the rotor is measured on a roundness measurement machine in our metrology lab and used in the orbit model.

### 2.1.1 Flow through the porous material

From the continuity equation, the flow through the porous medium can be modelled, assuming Darcy's law, as:

$$\frac{-K}{2\mu\phi} \nabla^2 p' + \frac{\partial p'}{\partial t} = 0$$

with  $p'$  the pressure in the porous material,  $K$  the permeability of the material [ $m^2$ ],  $\mu$  the dynamic viscosity of air [Pa.s] and  $\phi$  the effective porosity [ ].

### 2.1.2 Flow in the bearing clearance

The differential equation for the pressure in the bearing clearance of a porous journal bearing can be derived from the Navier-Stokes equations, neglecting the inertia forces compared to the viscous forces. This results in the Reynolds equation. Accounting for the effect of velocity slip at the boundary-layer of the porous wall – fluid film interface as shown in [3], the Reynolds equation can be written as:

$$\frac{\partial}{\partial x} \left( \frac{ph^3}{12\mu} \frac{\partial p}{\partial x} (1 + \Phi) \right) + \frac{\partial}{\partial y} \left( \frac{ph^3}{12\mu} \frac{\partial p}{\partial y} (1 + \Phi) \right) - \frac{u_s}{2} \frac{\partial ph}{\partial x} (1 + \Psi) = \frac{K}{2\mu} \frac{\partial p'^2}{\partial z} \Big|_{z=h_p} + \frac{\partial ph}{\partial t}$$

where the dimensionless parameters  $\Phi$  and  $\Psi$  are expressed as:

$$\Phi = \frac{3(SH + 2\alpha^2 S^2)}{H(S + H)}, \quad \Psi = \frac{S}{S + H}$$

with  $\alpha$  the effective slip coefficient and  $S$  the slip parameter which is defined as:

$$S = \frac{\sqrt{K}}{\alpha c}$$

### 2.1.3 Calculation procedure

By solving both equations simultaneously using a semi-implicit method, namely the ADI-method owing to its computational efficiency and numerical stability, the pressure distribution  $p$  in the bearing clearance can be calculated for each time step  $\Delta t$ . Integration of the pressure distribution over the bearing surface as:

$$W_x = -p_a L \int_0^{2\pi} P \cos(\theta) r d\theta$$

$$W_y = -p_a L \int_0^{2\pi} P \sin(\theta) r d\theta$$

gives the film forces which acts on the rotor. Finally, using the equations of motion, the displacement of the rotor can be calculated for each angular position of the rotor.

## 3 Experimental validation

The trajectory of the rotor centre of an aerostatic porous journal bearing is analysed for a rotational speed of 60 rpm, as shown in Fig. 2. The corresponding radial error motion is 3 nm. These very encouraging results are validated experimentally by measuring the radial error motion of an aerostatic rotary table with porous journal bearing. The measurement setup and rotary table under test is illustrated in Fig. 3. The least squares synchronous radial error motion of the rotary table is 9 nm, as depicted in the polar plot of Fig. 4.

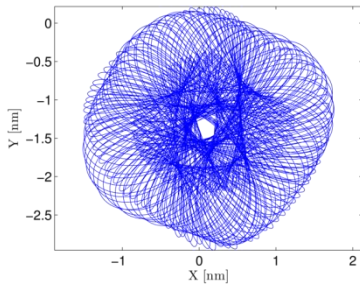


Figure 2: Trajectory of the rotor centre during one revolution calculated with the orbit model (3 nm).

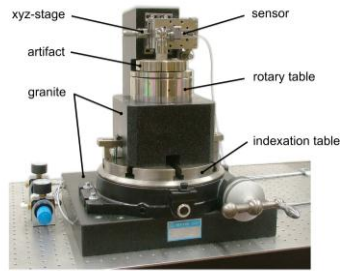


Figure 3: Measurement setup measuring the radial error motion of the aerostatic rotary table.

#### 4 Conclusions

In this work, a 2D-orbit model is proposed and validated for analysing the radial error motion of an aerostatic porous journal bearing. The radial error motion of an axis of rotation system is reduced to 9 nm by improving the feeding system, i.e. porous compensation. Experimental tests agree on the nanometre level with the orbit model.

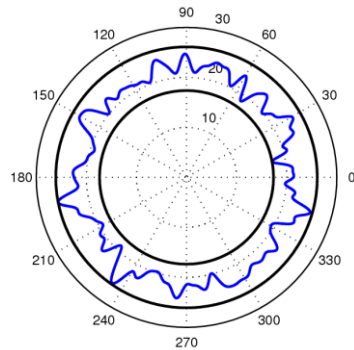


Figure 4: Radial error motion of the rotary table under test (9 nm).

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#### References:

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