

# A Family of Flexures That Eliminate Underconstraint in Nested Large-Stroke Flexure Systems

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## Abstract

In this paper we introduce a family of flexure linkages that may be used to eliminate underconstraint in nested, large-stroke flexure systems. Typically such systems achieve their large ranges of motion (i.e., stroke) because they consist of nested modules that possess redundant degrees of freedom (DOFs). Unfortunately, however, these redundant DOFs result in underconstrained bodies that are susceptible to parasitic errors, unwanted vibrations, and poor controllability. The flexure linkages proposed here eliminate underconstraint by coupling these bodies in a way that improves the system's static and dynamic performance without increasing its size or decreasing its stroke. Thus, such linkages impact the design of precision motion systems that traverse large ranges with appreciable speeds in potentially dynamic environments. Examples of such systems include energy harvesters, positioning stages, and MEMS devices such as resonators, micro-mirrors, and accelerometers. In this paper, we provide example flexure linkages as well as general guidelines for synthesizing them.

## 1 Introduction

Although flexure systems can achieve impressive repeatability and resolution, two of their main challenges are that (i) they cannot achieve a large range of motion compared with their overall size and (ii) their motions are generally accompanied with unwanted parasitic errors. These challenges are most commonly addressed by nesting opposing flexure modules inside of one another in a serial configuration. Consider, for instance, the parallel linear guide system in Fig. 1A. The stage of this system is limited to move a small fraction of its flexures' length and it will follow a slightly arcuate path instead of the desired pure translation. If, however this system is

nested in series with itself as shown in Fig. 1B, it remains the same size but its stage can move twice as far because both rigid bodies within the system possess a redundant translational DOF. Moreover, the opposing arcuate motions of the system's stage and intermediate body cancel as long as their displacement ratio is 2:1. Thus this system's stage can achieve a pure translation with minimal parasitic error. Although nesting a flexure system in this way addresses the issues of range and parasitic error, it also introduces other challenges. For example, the redundant DOFs of both rigid bodies cause the system to be underconstrained (i.e., the intermediate body is not fully constrained and is thus free to move with its redundant DOF when the stage is held fixed with respect to the system's fixed ground). System's that are underconstrained are susceptible to unwanted vibrations and possess mode shapes with natural frequencies that are comparable with the natural frequencies of the mode shapes associated with their desired DOFs. Thus, underconstrained systems are difficult to control even using closed-loop approaches. Furthermore, the stage-to-intermediate-body-displacement ratio necessary to eliminate the parasitic error of such nested systems are not enforced passively. As the stage from Fig. 1B translates forward, its intermediate body has a tendency to pull back dropping the system's off-axis stiffness substantially and displacing the stage with an unwanted parasitic error.

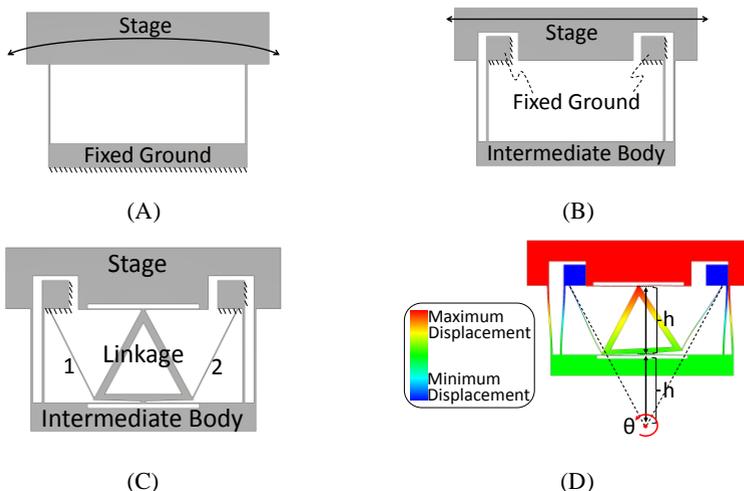


Figure 1: Linear guide system (A). Nested, large-stroke version (B). Linkage added (C). Stage and intermediate body are linked with a 2:1 displacement ratio (D).

The purpose of this paper is to introduce a new family of flexure linkages that address these issues while allowing such nested systems to maintain their size and large stroke capabilities. These linkages eliminate the underconstraint inherent to such systems by coupling their stages with their intermediate bodies in such a way that forces them to move with displacement ratios that eliminate parasitic error. They also enable designers to independently increase the natural frequencies of unwanted mode shapes and, thereby, significantly improve dynamic behaviour and controllability.

## 2 Flexure Linkages That Eliminate Underconstraint

A linkage that eliminates the underconstraint within the system of Fig. 1B is shown in Fig. 1C [1]. The flexures labelled 1 and 2 guide the linkage's rigid body with a rotation about an axis centred at the intersection point of their dashed lines of action (Fig. 1D). As the linkage rotates  $\theta$ , the stage translates  $2h\theta$  and the intermediate body translates  $h\theta$ . Thus, this symmetric linkage maintains the system's size and stroke while eliminating underconstraint and forcing the desired displacement ratio (i.e., 2:1) necessary to minimize parasitic error. An analogous rotary system is shown in Fig. 2A. Its analogous linkage consists of a rigid body that translates instead of rotates (Fig. 2B). When it translates a distance  $d$  the system's stage rotates  $d/h$  radians and its intermediate body rotates  $d/2h$  radians (Fig. 2C). Thus, the rotational stage-to-intermediate-body ratio is also 2:1. The previous two examples were planar systems. Suppose we wish to eliminate underconstraint in a spatial nested system like that shown in Fig. 2D. This system also possesses a single rotational DOF. A possible linkage that eliminates underconstraint in this system is shown in Fig. 2E. This linkage is tuned to enforce the desired rotational stage-to-intermediate-body ratio (2:1) as shown in Fig. 2F. It is also possible to eliminate underconstraint in multi-DOF systems like the one shown in Fig. 2G. This system possesses two translations shown as black arrows and an orthogonal rotation shown as a line with a circular arrow about its axis. A possible linkage that simultaneously enforces the desired displacement ratios for all three DOFs is shown in Fig. 2H. Its first two mode shapes (i.e., DOFs) are shown in Fig. 2I. Once designers appropriately nest flexures to achieve desired stroke-to-size requirements, they may then utilize the transmission-based FACT synthesis approach [2] to synthesize various linkage designs that eliminate underconstraint and set desired displacement ratios for any set of DOFs.

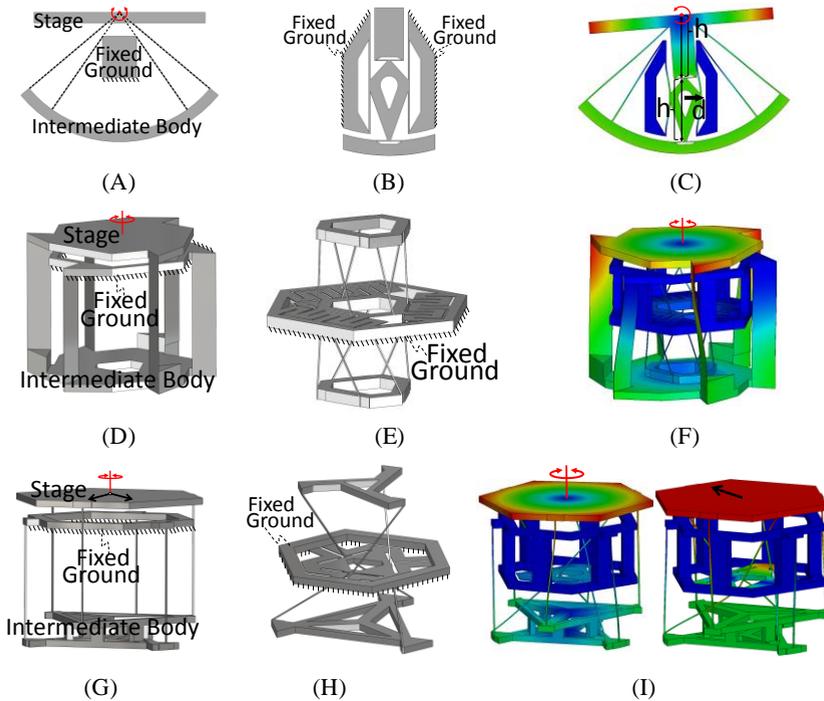


Figure 2: Nested, large-stroke systems (A), (D), and (G). Linkages (B), (E), and (H). DOFs appropriately linked with desired displacement ratios (C), (F), and (I).

## Conclusion

A family of flexure linkages that eliminate underconstraint in nested, large-stroke flexure systems has been provided. Guidelines for synthesizing such linkages for spatial multi-DOF systems have also been provided. This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344. LLNL-PROC-617832.

## References:

- [1] Panas, R.M., Hopkins, J.B., Szklarzewski, V., Schanne, E., "Eliminating Underconstraint in Double-parallelogram Flexure Mechanisms," submitted to *Prec. Eng.*, January 2013.
- [2] Hopkins, J.B., Panas, R.M., 2013, "Design of Flexure-based Precision Transmission Mechanisms Using Screw Theory," *Prec. Eng.*, 37: pp. 299-307.