An Approach to the Optimal Observer Design with Selectable Bandwidth

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Abstract

It is well know that the Kalman filter performs the best possible state estimator for processes affected by noise. A consequence of the optimality, is that the convergence speed of the estimation error cannot be selected by the user, because it depends by the covariance of the noises. Since for several application this characteristic could be undesired, this paper introduces an alternative design approach, in which the state estimation error could be optimized for a given bandwidth by the user. The effectiveness of the method is shown with an illustrative example.

1 Introduction

The classical full order observer for a linear system (A,B,C,D) is defined as follows

$$dx_s/dt = Ax_s(t) + Bu(t) + L(y(t) - Cx_s(t) - Du(t)) \qquad \text{(Equation 1)}$$

where: x_s are the estimations of the real states x, u and y are the input and the output of the system respectively, and the matrix L weights the correction given by the deviation between the estimated output by the observer and the real output of the system. Usually the two following methods can be used to determine the matrix L: the matrix can be selected in order to obtain a desired convergence speed of the estimation error x_s-x or, alternatively, the matrix L can be determined in order to minimize the variance of the estimation error $E[(x_s-x)^2]$ by using the well know Kalman-Bucy filter described in the seminal papers [1]. The first way allow the user to decide the bandwidth of the estimation, but does not guarantees the optimality of the estimation in presence of noises in the observed systems, counter-wise , the second way, guarantee the optimality of the estimation but does not directly allow the user to select the observer bandwidth. This paper introduces a third simple approach to the observer design, in which for a given bandwidth decided by the user, the optimal observer estimator is derived.

2 Alternative observer design method

Firstly the method will be introduced for multi output system, the case for single output systems will be discussed later. In the case in which the system possesses $m>1$ outputs, the L matrix results of dimensions n x m. It is well known, if the system is observable, that to place the n observer poles in a desired position, n free parameters are required. Consequently, the matrix L performs (n x m)-n supplementary degree of freedom that can be used for other purposes. The main idea presented by this paper, consists in the use of the supplementary degree of freedom performed by matrix L, to minimize the variance of the state estimation. More formally speaking, the following mathematical problem has to be solved

$$\min_L E[(x-x_s)^2] \text{ subject to } \det(\lambda_i I-A+LC)=0 \text{ for all } i \in\{1,n\},$$

where λ_i are the desired observer poles and n is the order of the system. A possible brute-force search solution to the problem above will be introduced in the illustrative example in the next paragraph; a more elegant solution to the problem will be subject of further papers. For single output systems, the number of parameters is insufficient to contemporaneously place the observer poles at a given position and simultaneously minimize the variance of the estimation error, in this case the problem can be solved by relaxing some conditions on the observer poles. An example could be the following: impose the pole Euclidian norm of the poles but not the angle, obtaining consequently, some free parameters in the matrix L that can be used to minimize the variance of the estimation error.

3 Illustrative example

Problem set-up:

The state vector $x(t)=[x(t),v(t)]$ of a classic mass damper system in figure 1 has to be estimated.

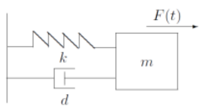

figure 1

The dynamic equation is

$$ma(t) = F(t) - kx(t) - dv(t)$$

here $a(t)$, $x(t)$ and $v(t)$ represent the acceleration, the position and the speed of the mass m. $F(t)$ is the input force, k and d are the spring and damping factor, respectively. The parameters of the system are:

$$m=0.056\ \text{kg},\ k=840\ \text{N/m},\ d=0.18\ \text{Ns/m}.$$

The input force amplitude $F(t)$ is known and the measurement of the position and of the acceleration are available, i.e. $y(t)=[x(t),\ a(t)]^T$. The input signal and the measurements are affected by white Gaussian noises with covariance matrices

$$C_U=4.47\text{x}10^{-8}\ N^2 \text{ and } C_Y=\text{diag}([5.08\text{x}10^{-16}\ \text{m}^2,.4.47\ \text{x}10^{-6}\ \text{m}^2/\text{s}^4]).$$

For control purposes, the bandwidth of the observer has to be 600 rad/s.

Solution:

Since the bandwidth has to be 600 rad/s, the following observer poles can be selected: λ_1 =-548 rad/s, λ_2 =-670 rad/s. To determine the matrix L according to the principle introduced by this paper, several procedures can be used; in this case the following method has been applied. Since the order of the system is 2 and the output of the system are 2, the matrix L result to be square composed by 4 elements, i.e.

$$L = [l_{11}\ ,\ l_{12}\ ;\ l_{21}\ ,\ l_{22}].$$

In order to reduce the problem complexity, the task has been solved as follows. Since one of the measurements (the acceleration a), corresponds to a derivative of a state variable (the speed v), the parameters of L have been used to express the estimated acceleration, i.e. the second element of the vector dx_s/dt, as a weighted sum of the acceleration determined using the model of the system called $a_{model}(t)$, and the measured acceleration $a(t)$, i.e.:

$$\text{estimated acceleration} = (1-w).\ a_{model}(t) + w.a(t),$$

where w is the weighting factor, i.e. a real number in the interval [0,1]. With this set-up, the coefficients l_{12} and l_{22} becomes 0 and w respectively, reducing by one the number of parameters for the covariance minimization. The obtained observer respecting the upper condition is:

$$dx_s/dt = (I-\Lambda)\ (A-LC)x_s(t) + (I-\Lambda).B.F(t) + L.y(t) + \Lambda.a(t)$$

where $\Lambda=\text{diag}([1,l_{22}])$. Then, the optimal value of the parameter l_{22} has been obtained by using a brute-force search in the interval [0,1], meanwhile the parameters l_{11} and l_{21} have been determined to place the poles in the desired position.

The illustrative example has been solved by using also the standard methods, i.e.: the pole placement method of Matlab, and by using the Kalman filter method [1]. All the results have been reported in table 1. The outcomes show that the Kalman filter version performs an observer with the better performance in sense of estimation noise, but with a bandwidth lower than the desired one. Contrariwise the pole placement algorithm of Matlab, performs a solution with the desired bandwidth but with a not optimized noise. Instead, the suggested concept performs a compromise between the two classical solutions.

Method	**Obtained bandwidth**	**Estimation RMS noise**
Standard pole placement	600 rad/s	5.63×10^{-4}
Kalman	285 rad/s	1.72×10^{-8}
Proposed method	600 rad/s	3.36×10^{-8}

Table 1 : results

The proposed procedure can be generally applied when some measurements correspond to the derivative or some states. We are working for a more general and elegant method to determine the matrix *L*, according to the idea proposed in this paper. This topic is the subject of our current researches.

4 Conclusions

The presented observer design method allows an optimal observer design in presence of bandwidth constrains.

References:

[1] R. E. Kalman and R. S. Bucy, “New results in linear filtering and prediction theory,” *Trans. ASME—J. Basic Eng.*, vol. 83, pp. 95–108, 1961, ser. D.