FEM model based POD reduction to obtain optimal sensor locations for thermo-elastic error compensation

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Abstract

To compensate thermo-elastic deformations in precision systems Error Compensation Models (ECM) can be used that predict thermo-elastic deformations based on measured temperatures. This thermal ECM is basically a matrix representing the relation between the temperature readings of a number of sensors on a structure and the position shift of the point(s) of interest. An accurate ECM requires determination of the right number of temperature sensors and the selection of good temperature sensor locations. For this, dominant temperature shapes describing the thermal behaviour of the system can be employed. These dominant temperature shapes result from a Proper Orthogonal Decomposition (POD) of temperature data obtained from simulations with a FEM model using time dependent loads. A new algorithm is proposed to select sensor locations from all nodal locations in the FEM model that can properly identify the set of dominant temperature shapes. Furthermore, a reduced FEM model approach is used to make POD decomposition practically possible as well as frequency domain evaluation of the thermo-elastic ECM.

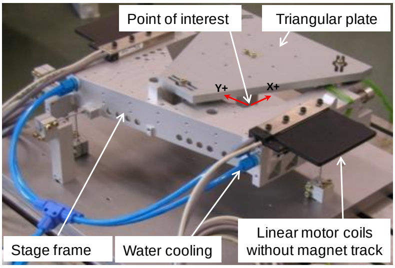

Fig.1 Precision stage test set-up

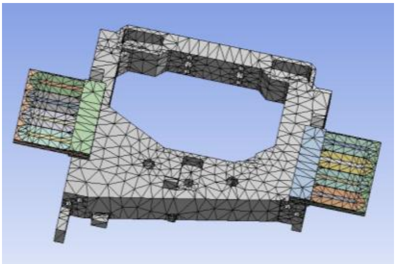

Fig.2 FEM model stage frame and motors

1 Introduction

The work has been applied on a precision stage that basically consists of a stage frame and two linear motors connected to the side of the frame (see figure 1). Furthermore, a triangular plate is mounted on the frame. In the middle of the triangular plate a rectangular bar is connected which is pointing downwards. The end of the bar is the point of interest. The stage frame is conditioned with cooling water running through two channels on both sides of the frame.

2 FEM model based POD reduction

The FEM model that has been made of the stage application is limited to the stage frame with its linear motor coil assemblies (see figure 2). The model is used to calculate the temperatures as function of time with known motor loads. A temperature identification matrix is generated from all nodal temperatures at each time instant. This identification matrix can be decomposed as the product of three other matrices using POD also known as SVD (Singular Value Decomposition). The columns of the first matrix U_T are *m* independent orthogonal POD temperature shapes. The second matrix Σ_T is basically a diagonal matrix with Singular values indicating the importance of each POD shape. Since obtaining the temperature identification matrix for all nodes, and decomposing it using the POD algorithm is very demanding in terms of computation time and memory, the FEM model has been reduced using Arnoldi reduction [2]. This made it possible to solve the thermal problem with only 120 Arnoldi states X and still obtaining results which are accurate for load fluctuation up to 1 Hz. The Arnoldi states X and FEM model temperatures T are related by the Arnoldi projection matrix V: $T \approx \mathbf{V} \cdot X$.

Now an Arnoldi state identification matrix is calculated using the reduced model. This matrix is decomposed resulting in the matrices: U_X, Σ_X and W_X. Here the shape matrix U_X is of dimensions pxp with p the number of Arnoldi states. It can be proven that the first p POD temperature shapes of U_T can be approximated by: $\tilde{\mathbf{U}}_T = \mathbf{V} \cdot \mathbf{U}_X$.

From the POD temperature shapes $\tilde{\mathbf{U}}_T$ the most relevant shapes, with the largest Singular values, are selected. Furthermore, the displacement sensitivity of the point of interest for thermo-elastic deformations of each relevant POD temperature shapes is investigated by enforcing the POD temperature shapes as a prescribed temperature condition on the FEM model and calculating the deformations. The overall shape importance can be expressed by the product of Singular value and shape sensitivity.

3 Sensor placement

Next the proper sensor locations need to be determined to identify the selected POD temperature shapes. We want to select the sensor locations from all nodal locations of our FEM model. As a criterion to optimize the sensor locations we use the condition number of the matrix formed by the coefficients of the selected POD shapes at the nodes of the selected sensor locations (see [1] for details). This criterion helps to come up with an ECM with relatively small coefficients which limits the effect of temperature measurement inaccuracies and sensor noise. Calculating the condition numbers for all *m* possible (nodal) sensor locations is practically impossible due to the combinatorial nature of the problem, so an efficient algorithm is necessary.

The algorithm described in [3] focuses on modal shape identification of structural dynamics. Although the algorithm seemed to be appropriate for our problem, it did not yield very good results in our case. In order to solve this problem we propose a new algorithm that determines sensor locations resulting in a low condition number.

3.1 Algorithm

The temperatures of each POD temperature shape are sorted with respect to their absolute value. The FEM nodes are sorted accordingly. This results in a matrix of node numbers of which the columns are the sorted nodes of each POD shape. For the nodes in each row the condition number is calculated. The nodes in the row with the smallest condition number are selected as sensor locations.

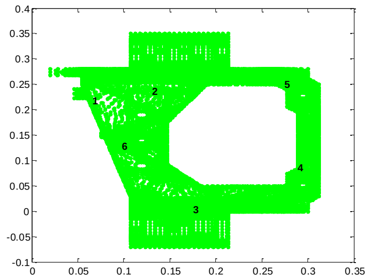

Fig.3 Selected sensor locations

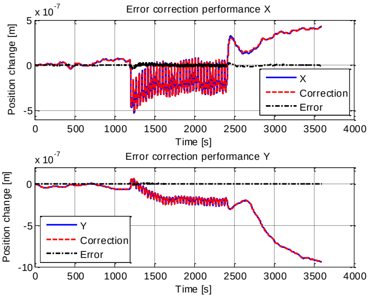

Fig.4 Correction model performance

The algorithm has been applied on a subset of nodes limited to the frame surface. This resulted in a sensor location set (see figure 3) with a low condition number.

4 Error Correction Model

The ECM is derived by means of a least square fit applied on simulated time dependent temperature data at the selected sensor locations and the corresponding displacements at the point of interest. Note, that also measured data can be used. Next, the predicted XY-position changes of the frame are calculated and compared against the simulated XY-position changes (see figure 4). This shows small differences of less than 8% for X-position changes and less than 1% for Y-position changes. A more general approach to evaluate the ECM is to investigate the performance in the frequency domain. Figure 5 shows the frequency response function (FRF) for the motor load variation. The FRF shows that the ECM (--) performs reasonable well for motor load variation over the whole frequency range of interest. Similar, the performance w.r.t. cooling water temperature (see figure 6) and ambient temperature variation can be evaluated.

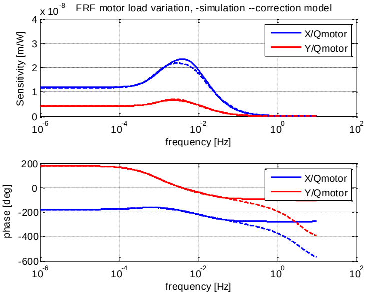

Fig.5 Frequency response to motor load

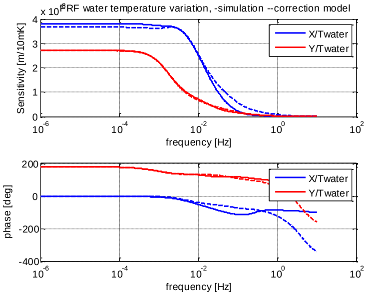

Fig.6 Frequency response to water temp.

References:

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[3] C. Stephan, 'Sensor Placement for Modal Identification', Mechanical Systems and Signal Processing 27, (2012) 461-470