# Geometrical-based approach for flexure mechanism design

T.J. Teo[1], G. Z. Lum[1,2,3], G.L. Yang[1], S. H. Yeo[2], M. Sitti[3]

[1]*Singapore Institute of Manufacturing Technology, Singapore*
[2]*Nanyang Technological University, Singapore*
[3] *Carnegie Mellon University, United States.*

*tjteo@SIMTech.a-star.edu.sg*

**Abstract**

This paper introduces an alternate approach for designing a flexure-based parallel mechanism (FPM). It involves a systematic design methodology that couples classical kinematics with modern geometrical optimization techniques. At sub-chain level, a novel topological and structural optimization technique is introduced to synthesize and optimize the geometry of the joint/limb based on desired stiffness characteristics. At configuration level, the moving masses and stiffness of the entire FPM are optimized based on desired dynamics. Using this new design approach, the system characteristics of the FPM is optimal and deterministic. This paper presents how this geometrical-based approach was used to design a 3-axes planar motion FPM.

## 1 Introduction

For many years, an exact constraint method is a well-established kinematic approach for designing any flexure-based joint/mechanism [1-2]. Even approaches introduced lately, e.g., the Freedom and Constraint Topology (FACT) [3] and those derived from screw theory [4] etc., are variants of the exact constraint method. These approaches have their merits when the constraints are ideal and the size of the synthesized mechanism is unlimited. Yet, they could only synthesis the topology of a mechanism based on ideal constraint conditions rather than delivering an optimal design based on desired system dynamics. This paper presents systematic design methodology, which couples classical kinematics with modern geometrical optimization techniques, to design an optimal FPM based on desired stiffness characteristics and moving masses.

## 2 Systematic design methodology

The first step of the design methodology is to synthesize the type of parallel-kinematic configuration based on the desired degrees-of-freedom (DOF) and task etc. At sub-chain level, a novel topological optimization is proposed to synthesize the flexure-based joints/limbs of the selected configuration. Concurrently, the structure optimization delivers the optimal geometries based on given stiffness characteristics. At configuration level, the moving mass of each joint/limb is predicted through mass condensation method. Hence, both stiffness and moving masses of the entire FPM can be optimized based on the desired workspace and size constraint. In this paper, design of a planar motion FPM is used to demonstrate this proposed methodology.

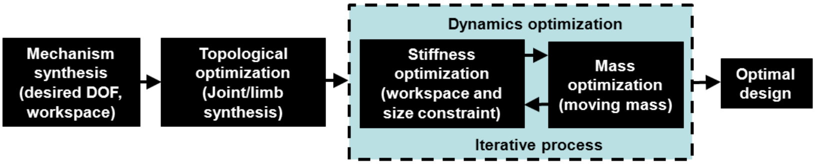



Figure 1: Block diagram representation of the systematic design methodology.

## 2.1 Mechanism synthesis

To achieve a 3-DOF planar motion, i.e., X-Y-θz, 3RRR, 3PRR, and 3PPR [5] are possible parallel-kinematics configurations (prismatic; P and revolute; R). 3PPR was chosen as the compliant P joints are more deterministic than the compliant R joints. The schematic of 3PPR is shown in Fig. 2 where the moving platform is connected to the fixed base by three identical parallel chains. Each chain comprises of a serially-connected active P joint and a passive RP joint. The overall size is 300x300mm$^2$ to amplify the errors for proper evaluation while the workspace is targeted at 4mm$^2$ x 2°.

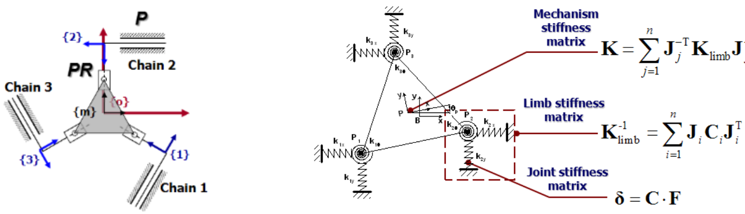



Figure 2: Schematic representation of 3PPR and its stiffness modelling.

## 2.2 Topological optimization: Mechanism-based approach

In this work, a hybrid topological and structural optimization technique is used to deliver optimal designs for the joints. Termed as mechanism-based approach, elementary kinematics chains are used as basic genes for the joint optimization, which runs on Generic Algorithm. Here, a generic 4-bar kinematics chain is used to synthesize the 1-DOF P joint and a generic 5-bar kinematics chain is used for 2-DOF

PR joint synthesis. For P joint, $C_{11}$ needs to be very compliance compared to other components of the compliance matrix. Hence, the objective function is formulated as

$$\min\left(F_1(\mathbf{x}) = \left|\frac{\prod_{i=2}^{6}\prod_{j=1}^{i} C_{ij}}{C_{11}^{\ 20}}\right|\right) \quad \text{subjected to} \quad \begin{matrix}\mathbf{K}_s\mathbf{u}_s = \mathbf{f} \\ \mathbf{h}_2(\mathbf{x}) \le 0\end{matrix} \qquad (1)$$

For PR joint, both $C_{11}$ and $C_{66}$ need to be very compliance compared to other components of the compliance matrix. Thus, the objective function is formulated as

$$\min\left(F_2(\mathbf{x}) = \left|\frac{\prod_{i=2}^{6}\prod_{j=1}^{i} C_{ij}}{C_{11}^{\ 19}C_{66}^{\ 19}}\right|\right) \quad \text{subjected to} \quad \begin{matrix}\mathbf{K}_s\mathbf{u}_s = \mathbf{f} \\ \mathbf{h}_2(\mathbf{x}) \le 0\end{matrix} \qquad (2)$$

Subsequently, optimizations are conducted based on these objective functions. The evolutions from the basic genes (kinematic chains) to optimal joint designs in both topological and structural forms are shown in Fig. 3.

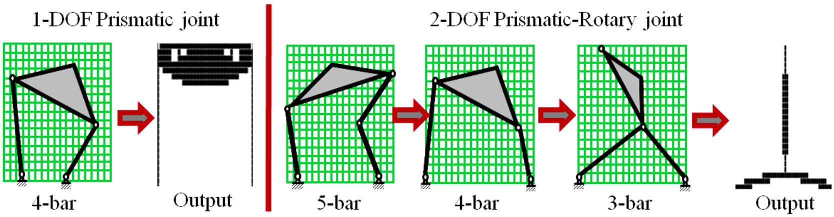
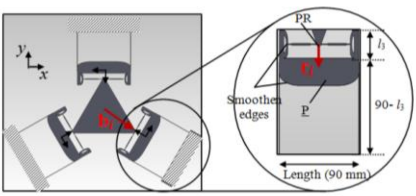



Figure 3: Concurrent evolution of topology and structure for both P and PR joints.

### 2.3 Configuration level: Overall mass and stiffness optimization

At configuration level, optimization constrains are based on desired workspace and size constraint. Using the compliance matrices derived from the optimal joints, the stiffness of the FPM can be obtained through kinematic stiffness modelling (Fig. 2).



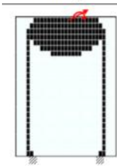

| Design | $\omega_n^2$ by FEA | $\omega_n^2$ predicted |
|---|---|---|
|  | $1.32\times10^7$ | $1.33\times10^7$ |
| | $5.28\times10^7$ | $5.28\times10^7$ |
| | $1.44\times10^8$ | $1.63\times10^8$ |
| | $1.52\times10^9$ | $2.05\times10^9$ |
| | $4.77\times10^9$ | $1.02\times10^{10}$ |
| | $5.86\times10^9$ | $3.83\times10^{11}$ |

Figure 4: Others optimization parameters

Figure 5: Mass prediction.

At this stage, only the stiffness of the proposed FPM was optimized with the aim of maximizing all non-actuating stiffness while minimizing all actuating stiffness through parameters such as the flexure length in P joint and the base of the PR joint (Fig. 4). In future, the proposed dynamics optimization needs to be done concurrently with a new mass optimization algorithm; using mass condensation technique, which at this stage has proven to have good prediction on the first 3 frequency modes of optimal P joint design as shown in Fig. 5.

## 3 Experimental results

Stiffness evaluation is conducted using PICO-motor to induce load while the motion of the end-effector is captured by a non-contact 3D-GOM system. At this stage of research, only the actuating stiffness of the developed FPM are evaluated. Results plotted in Fig. 6 show that the actuating compliance in Y-axis and about Z-axis are $3.91\times10^{-5}$ m/N and 0.0156 rad/Nm respectively. Comparing with the theoretical prediction of $3.39 \times10^{-5}$ m/N and 0.0133 rad/Nm, such small deviations prove that this approach is good for designing FPM based on desired stiffness. The prototype has achieved positioning and angular resolutions of 50nm and 0.2 arcsec respectively throughout a workspace of $4\text{mm}^2 \times 2°$.

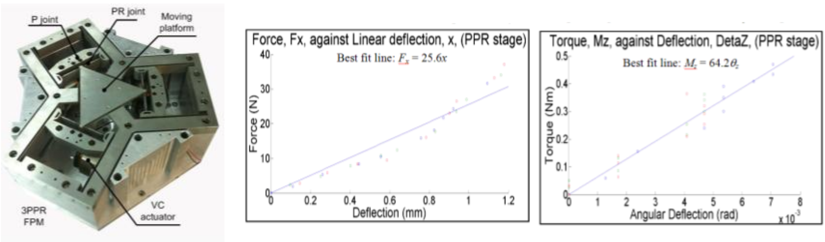



Figure 6: Developed prototype and measured compliance in Y-axis and about Z-axis.

## 4 Conclusion

This paper presented a geometrical-based approach to design a FPM. A novel hybrid topology optimization technique for stiffness optimization, and a new technique for mass optimization are introduced. Experimental results show that this approach is good for designing FPM based on desired system dynamics, i.e., stiffness and mass.

**References:**

[1] AH. Slocum, *Precision Machine Design*, Prentice-Hall, Inc.; 1992.
[2] DL. Blanding, *Principles of Exact Constraint Mechanical Design*, Kodak; 1992.
[3] JB. Hopkins, ML. Culpepper, Synthesis of precision serial flexure systems using freedom and constraint topologies (FACT), *Prec. Eng.*, 2011 (35), 638 – 649.
[4] JB. Hopkins, RM. Panas, Design of flexure-based precision transmission mechanism using screw theory, *Prec. Eng.*, 2013 (37), 299 – 307.
[5] GL. Yang, W. Lin, TJ. Teo, CM Kiew, "A flexure-based planar parallel nanopositioner with partially decoupled kinematic architecture," *Proc. of EUSPEN2008*, 18 – 22 May, Zurich, Switzerland, 2008, vol. 1, 160 – 165.