Singular Analysis of the Least Squares Estimation in the Measurement of Machine Tool Error using Ball-bar

D.M. LEE[1], K.I. LEE[1], S.H. YANG[1]
[1]*Kyungpook National University, Republic of Korea*

syang@knu.ac.kr

Abstract

Reverse kinematic analysis is essential to estimate parameterized geometric errors from ball-bar equation which shows the relation between geometric errors and measured ball-bar data. Parameters are calculated by linear least squares estimation but it suffers from singular problem which affect to estimation accuracy of parameterized geometric errors. In this study, a method is proposed to analyze singular problem directly by decomposing and re-combining column space at linear least squares estimation for ball-bar equation.

1 Derivation of ball-bar equation

The ball-bar consists of two precision balls and a sensor to measure the distance between two balls as shown in Fig. 1. During the installation of ball-bar, workpiece ball on workpiece table is located at a designed position $\mathbf{P}_c$ using tool cup located at the tool tip and tool cup is moved to designed position $\mathbf{P}$ away from workpiece ball at a distance equal to the nominal length R of ball-bar. And then, circular path is described to keep the distance between two balls as nominal length R.[1] But the actual positions $\mathbf{P}_c'$, $\mathbf{P}'$ and measured distance R+ΔR of two balls are deviated from nominal path and length, respectively, by volumetric errors $\Delta\mathbf{W}_c$, $\Delta\mathbf{W}$ consisting of geometric errors. In this case, the relationship between the positions of two balls and measured distance is given as Eq. (1) and ball-bar equation by eliminating higher order terms of volumetric errors in Eq. (1) is summarized as Eq. (2). Then, it can be represented as Eq. (3) to apply linear least squares method.

$$(R+\Delta R)^2 = \left\|\mathbf{P}' - \mathbf{P}_c'\right\|^2 = \left\|\mathbf{P} + \Delta\mathbf{W} - (\mathbf{P}_c + \Delta\mathbf{W}_c)\right\|^2 \quad (1)$$

$$R\Delta R = (\mathbf{P} - \mathbf{P}_c)^T (\Delta\mathbf{W} - \Delta\mathbf{W}_c) = \mathbf{N}(\Delta\mathbf{W} - \Delta\mathbf{W}_c) \quad (2)$$

$$\mathbf{b} = \mathbf{A}\mathbf{x} \quad (3)$$

Here, **A** is a matrix consisting of fitting function's components and **x** is a vector consisting of parameters. **b** is a vector consisting of measured data.

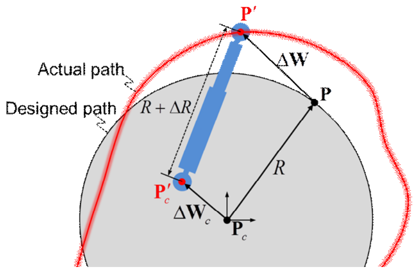

Figure 1: Configuration of ball-bar measurement

2 Analysis of singular problem

It is difficult to notice whether there are the singular problem or not at column space of parameters of geometric errors using Eq. (2) and to develop Eq. (3) by the complexity.[2,3] Therefore, volumetric error $\Delta\mathbf{W}$ is decomposed as Eq. (4) to analyze the singular problem directly. Here, **K** consists of the sign determined by the structure of machine tool and **F** means fitting function used to model the geometric errors. **C** consists of the parameters of the geometric errors. $\mathbf{D}_{ij}$, $\mathbf{E}_{ij}$ are vectors consisting of the components D_{ijk}, E_{ijk}(k=1,…,m) of mth orders fitting function for position error and angular error, respectively. $\mathbf{d}_{ij}$, $\mathbf{e}_{ij}$ refers to vectors consisting of parameters d_{ijk}, e_{ijk}(k=1,…,m) of geometric errors. Substituting Eq. (4) to Eq. (2), ball-bar equation is re-defined as Eq. (5). Using this method, the singular problem is easily analyzed without complexity, consideration of machine tool configuration and fitting function.

$$\Delta\mathbf{W}=\mathbf{KFC}=\left[\begin{array}{ccc|ccc}\mathbf{P} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{A}_z & \mathbf{A}_y \\ \mathbf{0} & \mathbf{P} & \mathbf{0} & \mathbf{A}_z & \mathbf{0} & -\mathbf{A}_x \\ \mathbf{0} & \mathbf{0} & \mathbf{P} & -\mathbf{A}_y & \mathbf{A}_x & \mathbf{0}\end{array}\right]\left[\begin{array}{ccc|ccc}\mathbf{D}_x & & & & & \\ & \mathbf{D}_y & & & \mathbf{0} & \\ & & \mathbf{D}_z & & & \\ \hline & & & \mathbf{E}_x & & \\ & \mathbf{0} & & & \mathbf{E}_y & \\ & & & & & \mathbf{E}_z\end{array}\right]\begin{bmatrix}\mathbf{d}_x \\ \mathbf{d}_y \\ \mathbf{d}_z \\ \mathbf{e}_x \\ \mathbf{e}_y \\ \mathbf{e}_z\end{bmatrix} \quad (4)$$

where,

$$\mathbf{P}=\begin{bmatrix} l_x \\ l_y \\ l_z\end{bmatrix}^T,\mathbf{A}_i=\begin{bmatrix} i_x \\ i_y \\ i_z\end{bmatrix}^T,\mathbf{D}_i=\begin{bmatrix}\mathbf{D}_{ix} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{iy} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{D}_{iz}\end{bmatrix},\mathbf{E}_i=\begin{bmatrix}\mathbf{E}_{ix} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E}_{iy} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{E}_{iz}\end{bmatrix},\mathbf{D}_{ij}=\begin{bmatrix} D_{ij1} \\ \vdots \\ D_{ijm}\end{bmatrix}^T,\mathbf{E}_{ij}=\begin{bmatrix} E_{ij1} \\ \vdots \\ E_{ijm}\end{bmatrix}^T$$

$$\mathbf{d}_i=\begin{bmatrix}\mathbf{d}_{ix} & \mathbf{d}_{iy} & \mathbf{d}_{iz}\end{bmatrix}^T,\mathbf{e}_i=\begin{bmatrix}\mathbf{e}_{ix} & \mathbf{e}_{iy} & \mathbf{e}_{iz}\end{bmatrix}^T,\mathbf{d}_{ij}=\begin{bmatrix} d_{ij1} & \cdots & d_{ijm}\end{bmatrix}^T,\mathbf{e}_{ij}=\begin{bmatrix} e_{ij1} & \cdots & e_{ijm}\end{bmatrix}^T$$

Finally, the ball-bar equation of Eq. (3) is re-arranged as Eq. (5) & Eq. (6).

$$\mathbf{N}\Delta\mathbf{W} = \mathbf{NKFC} = \begin{bmatrix} N_x\mathbf{PD_x} \\ N_y\mathbf{PD_y} \\ N_z\mathbf{PD_z} \\ \left(N_y\mathbf{A_z} - N_z\mathbf{A_y}\right)\mathbf{E_x} \\ -\left(N_x\mathbf{A_z} - N_z\mathbf{A_x}\right)\mathbf{E_y} \\ \left(N_x\mathbf{A_y} - N_y\mathbf{A_x}\right)\mathbf{E_z} \end{bmatrix}^T \begin{bmatrix} \mathbf{d_x} \\ \mathbf{d_y} \\ \mathbf{d_z} \\ \mathbf{e_x} \\ \mathbf{e_y} \\ \mathbf{e_z} \end{bmatrix} \quad (5)$$

$$R\Delta R = \mathbf{N}\left[\mathbf{KF} - (\mathbf{KF})_c\right]\mathbf{C} \equiv \mathbf{AC} \quad (6)$$

$$\mathbf{A} = \left[-(x-x_c)^2 \quad -(x-x_c)(y-y_c) \quad (x-x_c)(z-z_c) \quad -(x-x_c)(y-y_c) \quad \cdots\right] \quad (7)$$

Using Eq. (6) with geometric errors modeled as 1st orders polynomial as an example, singular problem between 2nd and 4th columns is analyzed directly as Eq. (7). It means the parameters corresponding to those columns cannot be calculated accurately.

3 Improvement of estimation accuracy

New approaches can be regarded to increase estimation accuracy of parameters. Polynomial and trigonometric function has to be used to model the geometric errors at a time to avoid singular problem.

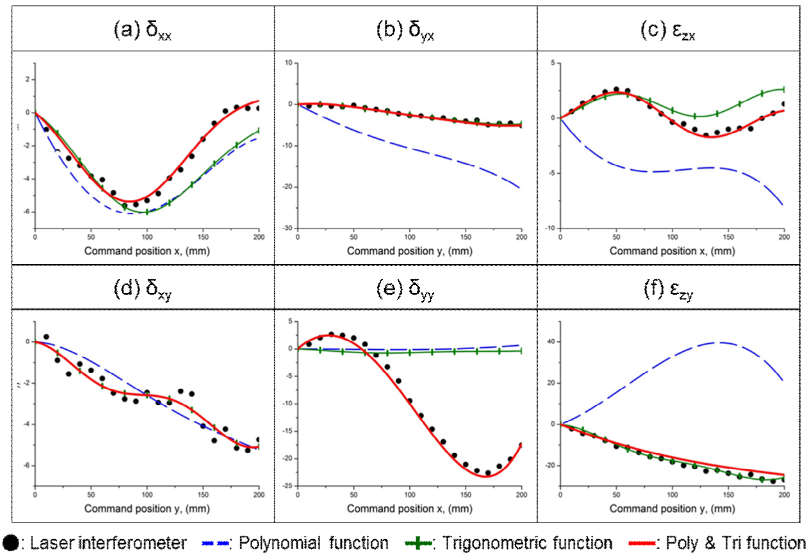

Figure 3: Singular problem by fitting method

The singular problem is inevitable at modeling the geometric errors only using n^{th} orders polynomial or trigonometric function. To test this approach, geometric errors of linear axis **X**, **Y** at machine tool are measured using laser interferometer and those geometric errors are estimated by using ball-bar equation and Eq. (6) as shown in Fig. 3. As expected, estimated geometric errors using 4^{th} orders polynomial or trigonometric function only shows large discrepancy by singular problem. On the contrary, geometric errors are well estimated compared with the result of laser interferometer when they are modeled using 4^{th} order polynomial and trigonometric function at a time.

4 Conclusion

In this study, singular problem at ball-bar measurement is analyzed to estimate geometric errors and the conclusions are summarized as follows:

1) The singular problems are analyzed directly by decomposing and recombining column space at linear least square estimation.

2) The estimation accuracy of geometric error parameters can be increased by combined modeling technique.

Acknowledgements

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (2011-0020445), (2011-0018392).

References:

[1] ISO 230-1, "Acceptance Code for Machine Tools Part 1: Geometry Accuracy of Machines Operated under No Load or Finishing Condition," ISO, 1992.

[2] Hai, N., Yuan, J. and Ni, J., "Reverse Kinematic Analysis of Machine Tool Error using Telescoping Ball bar," ASME, PED 68-1, 277-286, 1994.

[3] Pahk, H.J., Kim, Y.S. and Moon, J.H., "A New Technique for Volumetric Error Assessment of CNC Machine Tools Incorporating Ball bar Measurement and 3D Volumetric Error Model," Int. J. Machine Tools and Manufacturing, 37(11), 1583-1596.