Scaling down Coriolis-based mass flow rate sensing

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Abstract

A Coriolis mass flow meter (CMFM) measures the rate of fluid flow from the effect of forces induced by the Coriolis effect on a vibrating tube. This work considers scaling a CMFM to measure small mass flow rates. The effect of the tube dimensions on the measured response, the pressure drop and the resonance frequency is analysed. It is shown that these properties cannot be tuned independently by scaling the tube's dimensions, posing a fundamental limit on the performance of a small flow CMFM.

1 Background

Coriolis mass flow meters (CMFMs) have a high accuracy and repeatability [1]. The measurement is independent from other fluid properties like density, viscosity and thermal capacity. A single CMFM can measure both fluids and gasses and a continuous tube is the only wetted part. These advantages make CMFMs suitable for many applications in chemical, food, pharmaceutical and semiconductor industry. Commercial CMFMs can measure flow rates up to 36,000 kg/h [1], but it is hard to scale down a CMFM for small flow rates [2]. In recent years we managed to scale down the zero stability to 0.1g/h for a CMFM with a pressure drop of 1 bar at 100 g/h water flow [3]. Further research is aimed at scaling down the zero stability further. This work considers some fundamental limits in the scaling of a CMFM.

2 System overview

The main component of the meter is a flexible tube carrying the fluid. The tube is excited at one of its resonance frequencies. A fluid flow through the tube is forced to follow the oscillating motion, which introduces reaction forces on the tube by the Coriolis effect. These forces induce an additional motion in the tube, which is out-of-phase with the excited motion. Figure 1 shows the excited and Coriolis-induced motion [3]. The tube motion is measured at two positions, where the response to the excited motion and the Coriolis-induced motion differs, which introduces a phase-shift in the measured signals. This phase-shift is proportional to the mass flow rate.

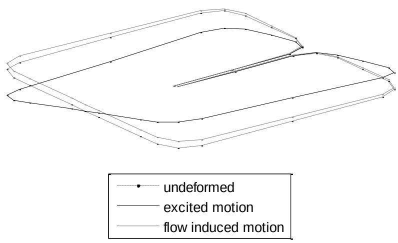

Figure 1: Tube motion

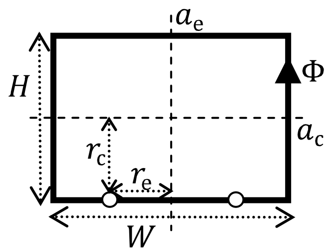

Figure 2: Tube model

The sensors, the tube and the actuation of the tube make a CMFM a mechatronic system. Scaling down a CMFM requires optimisation of all components and a well balanced integration thereof. The tube plays a central role in the performance of the CMFM as it affects the corrosion resistance, pressure-drop, maximum pressure, signal-to-noise ratio and sensitivity to external vibrations. In this work the influence of the dimensions of the tube on some essential CMFM properties is analysed.

3 Dimension analysis

The model in figure 2 is used to analyse the scaling effects. The tube is a rigid rectangular frame with dimensions W and H. The window can rotate about axes a_e and a_c, with associated inertias J_e and J_c and stiffnesses c_e and c_c. The damping of the motion is neglected. The fluid mass flow rate through the tube is denoted by Φ. The tube is excited with a sinusoidal periodic motion around the a_e-axis; $\theta_\mathrm{e} = \theta_\mathrm{e}\sin(\omega_\mathrm{e} t)$ with $\omega_\mathrm{e} = \overline{c_\mathrm{e}\ J_\mathrm{e}}$. This motion induces a Coriolis force in each of the straight sections of length W. The size of each force is $F_\mathrm{c} = W\Phi \times \theta_\mathrm{e} = \mathrm{W}\Phi\theta_\mathrm{e}\omega_\mathrm{e}\cos(\omega_\mathrm{e} t)$. These forces result in a rotation of the tube around the a_c-axis. The amplitude of this motion is derived from the folowing equation of motion and its periodic solution for F_c:

$$J_\mathrm{c}\theta_\mathrm{c} = \tfrac{1}{2}H2F_\mathrm{c} - c_\mathrm{c}\theta_\mathrm{c} \rightarrow \theta_\mathrm{c} = \frac{HW\Phi\theta_\mathrm{e}\omega_\mathrm{e}}{c_\mathrm{c} - J_\mathrm{c}\omega_\mathrm{e}^2}\cos\ \omega_e t \ \ = \theta\Phi\frac{HW}{\overline{J_\mathrm{c}c_\mathrm{c}}}\frac{\omega_\mathrm{c}\omega_\mathrm{e}}{\omega_\mathrm{c}^2 - \omega_\mathrm{e}^2}\cos\ \omega_\mathrm{e} t$$

The motion of the tube is measured at two points, located at r_e and $-r_\mathrm{e}$from the a_e-axis and r_c from the a_c-axis. The measured quantity m is the ratio between the amplitudes of the excited motion and the out-of-phase Coriolis-induced motion:

$$m = \frac{r_\mathrm{c}\theta_\mathrm{c}}{r_\mathrm{e}\theta_\mathrm{e}} = \frac{r_\mathrm{c}}{r_\mathrm{e}} \frac{\omega_\mathrm{c}\omega_\mathrm{e}}{\omega_c^2 - \omega_\mathrm{e}^2} \frac{HW}{\sqrt{J_\mathrm{c}c_\mathrm{c}}} \Phi$$

This equation shows that the measurement response signal can be enlarged by:

- increasing $r_\mathrm{c}/r_\mathrm{e}$, i.e. measuring close to the a_e-axis of the excitation and far from the a_e-axis of the Coriolis-induced motion.
- reducing $\omega_c^2 - \omega_\mathrm{e}^2$, i.e. taking the eigenfrequencies close together. It should be noted that for a small difference the damping becomes dominant in the response of the tube to the Coriolis force. This effect is not included in the analysis.
- increasing HW, i.e., increasing the size of the window.
- reducing $J_\mathrm{c}c_\mathrm{c}$, i.e., reducing the inertia and stiffness of the tube.

The Coriolis motion $r_\mathrm{c}\theta_\mathrm{c}$ associated with the zero stability of small-flow CMFMs is several nanometres only, illustrating the challange in constructing such CMFMs.

The scaling of the CMFM properties is considered in relation to the tube's diameter d and length L. The dimensions $W, H, r_\mathrm{c}$and r_e are assumed to scale with L. The tube's mass scales with d^2L and its inertia thus scales as $J{\sim}d^2L^3$. The tube's stiffness scales with $c{\sim}d^4L^{-1}$. The ratio of the eigenfrequencies is assumed to be independent of the tube dimensions. Substitution in the expression for the measurement yields:

$$m{\sim}Ld^{-3}$$

Thus, the measured Coriolis effect increases with dimension L while it reduces strongly with the diameter. Increasing dimension L and decreasing the diameter is not without consequence, because this increases the pressure drop ΔP along the tube. For small diameters the flow is laminar and the pressure drop can be computed from the Hagen-Poiseulle equation, with the following dependency on the length and diameter

$$\Delta P \sim Ld^{-4}$$

Another issue is the frequency of the tube, which should be sufficiently high to reduce the excitation of the tube by low-frequency external vibrations. The frequency scales with the tube's dimensions as:

$$\omega = \sqrt{c/J} \sim L^{-2}d$$

For a small-flow CMFM, the measured response should be increased to detect the flow rate accurately. The pressure-drop is allowed to scale equally, such that the flow rate for a certain pressure drop results in the same response. The relations above show that this leads to a reduction of the frequency, which makes the CMFM more sensitive to external vibrations. Increasing the freqeuency requires an increase of the pressure-drop or a decrease of the measured response. Thus the relations above give a fundamental limit on the CMFM properties due to scaling the tube dimensions.

In commercial CMFMs complex tube shapes are used and the inertia and stiffness are distributed along the tube. The presented scaling analysis is still applicable and quantities for the effective length, inertia and stiffness associated with the actuated and Coriolis-induced motion can be obtained from modal analysis and finite-element modelling [4]. The measured and modelled properties of a commercial CMFM range confirms the validity of the scaling analysis and the fundamental limit.

4 Conclusion

Scaling a CMFM for small mass flow rates requires optimisation of the tube, actuation and sensing elements and their integration. In this work the effect of the tube's dimensions on the meter performance is analysed. It is shown that the measured response, the pressure drop and the resonance frequency cannot be tuned independently, posing a fundamental limit on the meter's performance.

References:

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