Optimal Performance of a Controlled System with Structural Parameter Variation

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Introduction

Traditionally, sequential approaches to structural and control optimization are used to improve the performance of mechatronic systems. However, the chances of finding a truly optimal system can be increased if structure and controller are optimized in a simultaneous, integral optimization procedure. Our research is focussing on combining topology optimization and controller optimization. The first step towards an integral system optimization approach is to study the mutual dependence between structural and control parameters, and their combined effect on a chosen performance measure. Furthermore, sensor and actuator placement are important factors in achieving optimal system behaviour, and should be considered in combination with the specific dynamic behaviour. In this paper, we study the optimal design of a simplified motion control system. In particular, we focus on a fundamental understanding of the interdependence between sensor position, structural eigenmodes and controlled system performance.

1 Problem description

We consider the typical controlled system schematically depicted in Fig. 1. The mechanical system is depicted in Fig.2. It consists of a floating beam, a central actuator and a sensor. The beam is made from aluminium and has a width of 16 mm and a height of 30 mm. The length of the beam is a design variable and varies between 450 and 800 mm. The value of d, indicating distance between the middle of the beam and the sensor position, is fixed to 200 mm. This structure was modelled with 3D elastic beam elements and a state-space representation of the system was constructed, following the approach documented in [1].

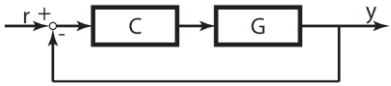

Figure 1: System, where G denotes a mechanical structure and C is a controller. r and y are a reference and an output of the system, respectively.

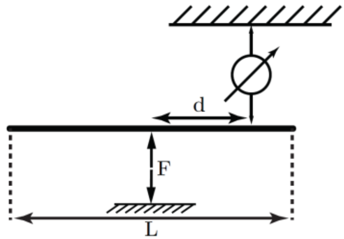

Figure 2: Mechanical structure.

2 PID Control and Optimization Objective

A PID controller is applied, which can be described as:

$$C(s) = k_p \left(1 + \frac{1+\tau_d s}{1+\frac{\tau_d}{N}s} + \frac{1}{\tau_i s} \right),$$

where k_p, τ_d and τ_i denote proportional, derivative and integral terms, respectively. The factor N defines the relation between pole/zero break-point frequencies for the differential term. We restrict ourselves to $N = 10$ and $\tau_d = 3\tau_i$ (see, e.g., [2]). Thus only two independent control parameters are left. As objective for the optimization, we consider the maximization of the open-loop gain at 1 Hz. This objective is characteristic for high-precision mechatronics machines. Such machines are typically designed for specifications based on their standstill or constant-velocity performance [3]. Practically, this means that the gain of the closed-loop system will be increased all over the frequency range. In addition, this also minimizes the positioning error. As constraints, we require a positive phase margin and a closed-loop frequency response of maximum 6 dB. These constraints guarantee the stability of the system and adequate responsiveness.

2.1 Analysis

The goal is to understand the interdependence between structural and control parameters, thereto for every given beam a controller is optimized. Note that integral optimization is not yet considered here. In this section, we examine the objective and constraint functions for a 600 mm long beam, by visualizing their dependencies on the control parameters τ_p and τ_d (both are scaled to be in range [0,10] by the scaling coefficients 10^4 and 10^{-3} , respectively). Fig. 3 shows that the objective function is quite smooth and monotonic, but the constraints (Fig. 4 and 5) are nonconvex and their feasible regions (unshaded areas) are disjoint. Transforming the problem into an unconstrained one using penalty functions reduced the sensitivity of the optimization outcome on the starting point. The gradient-free Nelder-Mead simplex method was used for optimization. Fig. 6 shows the optimum obtained by this procedure.

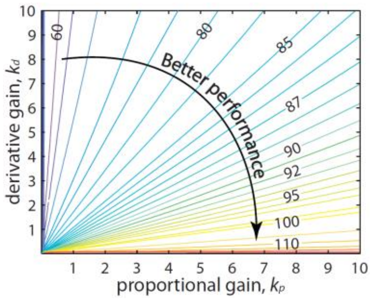

Figure 3: Objective: Open-loop gain at 1 Hz [dB].

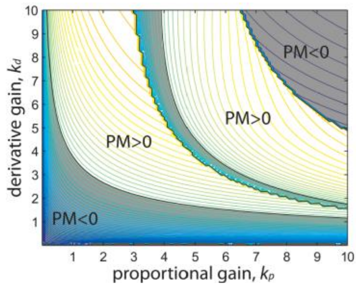

Figure 4: Constraint 1: Positive phase margin (PM>0).

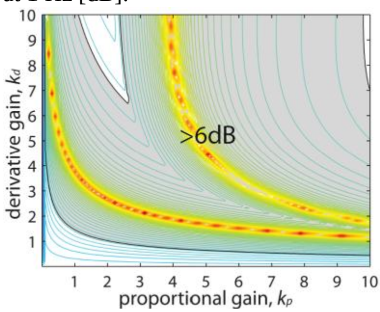

Figure 5: Constraint 2: Closed-loop frequency response < 6 dB..

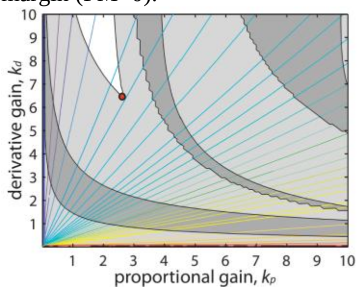

Figure 6: Combination of constraints, final feasible domain and optimum.

At which length should we expect a maximal gain? One would expect the best performance, when the sensor is collocated at the nodal point of the first eigenmode, where related vibrations become invisible for the controller. For the present problem, a nodal point will coincide with the sensor for L=725 mm. However, Fig. 7 shows that a maximum is achieved for L=710 mm. The difference is explained by the influence of higher frequency modes. When the sensor gets closer to the nodal point, the contribution of the second mode increases, while influence of the first one decreases. The optimization process finds a balance between these two effects.

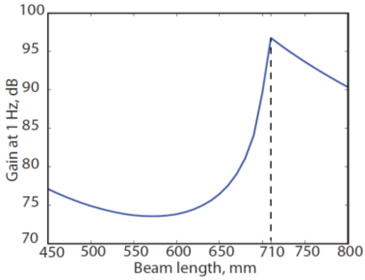

Figure 7: Optimal gains for different beam lengths with PID control.

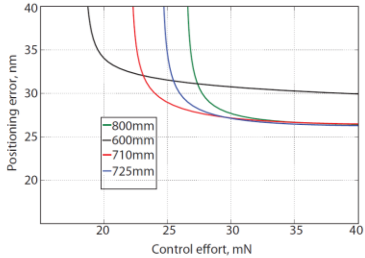

Figure 8: Pareto curves for different beam lengths with H_2 control.

3 H_2 control

As the next step, an H_2 controller is used. We follow the scheme, described in [3], to find an optimal balance between control effort and positioning error by means of Pareto curve. Furthermore, we use the same specifications for noises and disturbances as was done in the abovementioned work. Fig. 8 shows four Pareto curves for different beam lengths. It was found that the curve related to 710mm is closest to an ideal case (0,0), which means that in the considered problem the 710mm beam performs slightly better than 725 mm one, particularly regarding control effort.

Conclusions

- A simplified motion control system was studied for understanding the interdependence between structural and control parameters. A reliable optimization algorithm was then derived out of this study.
- For both controllers (PID and H2) the best performance of the system is achieved when the sensor does not coincide with the nodal point.

References:

[1] M.R. Hatch, Vibration Simulation Using MATLAB and ANSYS, Chapman and Hall/CRC, Boca Raton, USA, 2001.
[2] R.M. Schmidt, G.Schitter, J. van Eijk, The Design of High Performance Mechatronics, Delft University Press, Delft, The Netherlands, 2011
[3] Jabben L. Mechatronic Design of a Magnetically Suspended Rotating Platform. PhD Thesis Technical University of Delft, 2007.