An Iterative Learning Control Architecture For Precision Machining Processes

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Abstract
Machining processes, although presenting a cyclic behaviour, cannot exploit control strategies such as Iterative Learning Control (ILC) or Repetitive Control (RC) to improve tracking performance, since the initial position of the tool at the beginning of each cycle depends on the material removed during the previous cycles. This paper proposes a solution to this weakness by introducing a new control strategy that merges the ability of ILC to support the re-initialization of the initial conditions, with the ability of RC to support the dependencies of the initial conditions from previous cycles.

1 Introduction.
ILC and RC exploit the repetitive behaviour of a process, and successively reduce the control tracking error with each cycle. ILC is used to control systems that repeat a task with a re-initialization of the initial conditions at the beginning of every cycle [1]; thus standard ILC cannot support the dependence of the initial condition of a cycle from the trajectory run during previous cycles. RC deals with systems that cycle through the same task continuously without resetting the state variables [2], i.e. the initial condition of the current cycle is given by the end state of the previous cycle; thus standard RC control algorithms do not support the re-initialization of the initial condition for the new cycle. Machining processes such as milling, grinding, polishing and electro-discharge machining (EDM, see [3,4]) present a cyclic behaviour.
Unfortunately, these manufacturing processes cannot exploit ILC or RC to improve tracking performance since some process variables are reinitialized at the beginning of each cycle (like the horizontal position of the tool in milling), while others depend on the previous cycles (like the vertical position tool position, which must follow the receding tool surface).
The present work is a generalization of [3,4], and introduces a ILC design procedure suitable for a large spectrum of machining processes.

2 The model of a generic machining process

In this section the model of a generic machining process is determined. Without loss of generality, the motion of the system is restricted to occur along one axis. The position of the tool along this axis is denoted by the variable \( z(t) \) while \( z_p(t) \) is the position of the work-piece surface, and \( \delta(t)=z(t)-z_p(t) \) is the distance between the tool and the work-piece or the compression of the work-piece caused by the tool contact force. Then, the variable to be controlled can be modelled as an affine function of the gap \( \delta(t) \), i.e. \( y(t)=k_\delta \delta(t)+b \). In case of a the compression of the surface, \( y(t) \) can be the contact force and \( k_\delta \) the stiffness of the work-piece surface.

In the context of ILC, the lifted-domain representation is very convenient: a generic signal \( s(k) \) of length \( N \) is represented by a vector \( s=[s(0) \ldots s(N-1)]^T \) while a generic discrete-time linear time-invariant (LTI) system is represented by a matrix containing the values \( p_0, p_1, \ldots \) of the impulse response. Given the notation

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix},
\begin{bmatrix}
0 & 0 & \cdots & 1 \\
0 & 0 & \cdots & 1 \\
1 & \vdots & \ddots & \vdots \\
1 & \vdots & \ddots & \vdots \\
0 & \vdots & \ddots & 1 \\
0 & \vdots & \ddots & 0 \\
\end{bmatrix},
\begin{bmatrix}
1 & \cdots & 1 \\
1 & \cdots & 1 \\
1 & \cdots & 1 \\
1 & \cdots & 1 \\
1 & \cdots & 1 \\
0 & \cdots & 0 \\
\end{bmatrix},
\begin{bmatrix}
1 & \cdots & 1 \\
1 & \cdots & 1 \\
1 & \cdots & 1 \\
1 & \cdots & 1 \\
1 & \cdots & 1 \\
0 & \cdots & 0 \\
\end{bmatrix}
\]

the lifted-domain representation [2] of the machining process model is given by

\[
\begin{align*}
z_{j+1} &= P \cdot u_{j+1}, \\
\delta_{j+1} &= z_{j+1} - z_{p,j+1} \\
y_{j+1} &= k_\delta \cdot \delta_{j+1} + b + n_{j+1}, \\
z_{p,j+1} &= z_{A_j} + [0 1] \cdot z_{p,j}
\end{align*}
\]

The input \( z_{A_j} \) represents the receding movement of the machining surface during the \( j \)-th cycle, \( u_i \) is the reference position of the tool, \( n_i \) the input noise, and the matrix

\[
P = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ p_0 & 0 & \cdots & 0 \\ p_1 & \ddots & \ddots & \vdots \\ p_{N-1} & \cdots & p_1 & p_0 \end{bmatrix} \cdot (I - [1 0]) = \\
\begin{bmatrix}
1 & 0 & \cdots & 0 \\
1 - p_0 & p_0 & \cdots & \vdots \\
1 - p_1 - p_0 & p_1 & \ddots & \ddots \\
\vdots & \ddots & \ddots & \ddots \\
1 - \sum_{i=0}^{N-2} p_i & p_{N-2} & \cdots & p_1 & p_0 \\
\end{bmatrix}
\]

represents the dynamics of the axis closed-loop, where \( p_0, \ldots, p_{N-2} \) are the first \( N-1 \) samples of the impulse response of the process. This description includes the property of the system to give a constant output for a constant input [3].
3 ILC law

The ILC is described by the standard equation \( u_{j+1} = Q(u_j + L_e_j) \) with \( e_j = y_{ref,j} - y_j \), where \( L \) is the so-called learning function. \( Q \) can implement a low-pass filter \[3\], i.e.

\[
Q = \begin{bmatrix}
q_0 & 0 & \cdots & 0 \\
q_1 & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
q_{N-1} & \cdots & q_1 & q_0
\end{bmatrix} \cdot (I - \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}) = 
\begin{bmatrix}
1 & 0 & \cdots & 0 \\
1 - q_0 & q_0 & \ddots & \vdots \\
1 - q_1 & q_1 & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
1 - \sum_{i=0}^{N-2} q_i & q_{N-2} & \cdots & q_1 & q_0
\end{bmatrix}.
\]

The operator \( L \) is chosen as follows

\[
L \cdot e_j = l_1 \cdot \left( I - \frac{1}{N} \cdot [1] \right) \cdot e_j + \left( l_2 - l_3 \cdot \frac{1}{q-1} \right) \cdot \frac{1}{N} \cdot [1] \cdot e_j = L' \cdot e_j + l_3 \cdot 1 \cdot x_j,
\]

where \( 1/(q-1) \) denotes the integral operator, the proportional operator

\[
L' = l_1 \cdot \left( I - \frac{1}{N} \cdot [1] \right) + l_2 \cdot \frac{1}{N} \cdot [1],
\]

has been introduced, and the term

\[
x_{j+1} = x_j + \frac{1}{N} \cdot 1^T \cdot e_j,
\]

is related to the forward motion that the vertical axis undergoes during a cycle.

4 Closed-loop model and stability analysis

The state matrix of the controlled process is

\[
\Phi = 
\begin{bmatrix}
1 & -\frac{k_{\delta in}}{N} \cdot 1^T \\
l_3 \cdot 1 & P \cdot Q \cdot (P^{-1} - k_{\delta in} \cdot L)
\end{bmatrix}
\]

where the corresponding state is \([x_j \delta_j]^T\).

Comparison of the desired characteristic polynomial with the characteristic polynomial of \( \Phi \) yields the controller parameters

\[
l_1 = \frac{q_0 - \lambda_3}{k_{\delta} q_v p_0}, l_2 = \frac{2 - \lambda_1 - \lambda_2}{k_{\delta}}, l_3 = \frac{(1 - \lambda_1)(1 - \lambda_2)}{k_{\delta}}.
\]

The parameters \( l_2, l_3 \) determine the forward movement dynamics, and \( l_1 \) the cycle dynamic.

5 Controller design procedure

The controller design can be performed according to the following procedure

1) Identify the machining process parameters \( p_0, k_{\delta} \).
2) Choose the bandwidth of the low-pass filter and then determine \( q_0 \).
3) Determine the controller parameters $l_1, l_2, l_3$ with the equations set (1). Another controller design method, based on a least-square regression, uses a projection matrix $Q$ instead of the usual low-pass filter. Similar steps for the determination of the controller parameters hold [4].

6 Measurements

The controller has been implemented in an EDM (see results in Figure 1).

![Figure 1: z-axis behaviour with an integral controller (top) and ILC (bottom).](image)

Figure 1 depicts the electrode position above the work-piece surface. It can be noticed as the rough motions of the electrode present with the integral controller almost disappear with the ILC controller and as the gap distance become more constant.

6 Conclusions

The presented ILC improves the jitter and the tracking of machining process.

References: