Workspace and Dexterity Optimization of 3-PRR Planar Parallel Manipulator

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Abstract

With the advantages of high speed and carrying capacity, good dynamic performance, low cost and compact structure, parallel robots have a good prospect of application. In this work, the 3-PRR, a new planar parallel manipulator (PPM) is presented. First, the constraint features, position formulations of 3-PRR PPM will be introduced. Second, optimization design will be considered with 2 different methods: Monte Carlo Method and Global Condition Index. Monte Carlo Method is applied to determine the optimum parameter with the largest workspace. Based on Global Condition Index, parameter will be optimized with the best dexterity. Third, a multi-objective optimization problem will be finally proposed in order to determine optimum kinematic parameter with a required workspace and a better dexterity.

1 Introduction

In the parallel robot design, the key point is structural parameter selection, which directly affects the mechanism performance such as workspace and dexterity. Therefore, optimization design of the parallel structure parameters becomes a very important issue.

2 Design of 3-PRR Planar Parallel Manipulator

The planar parallel Manipulator has symmetric three identical PRR legs connecting from the fixed base to the end-effecter as shown in Fig 1. Each leg is of PRR configuration, with an active prismatic joint and two passive revolute joints. Each active prismatic joint is an actively-controlled actuator.

3 Optimization of Workspace for 3-PRR PPM

The optimization problem aims to determine the geometric parameters of the manipulator in order to maximize its workspace. To find the manipulator workspace, the entire possible workspace is introduced by a circle (Fig.2).

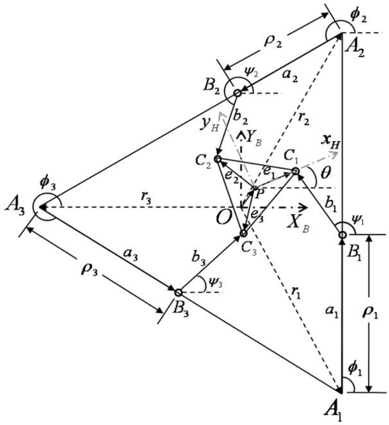

Fig.1: Schematic Diagram

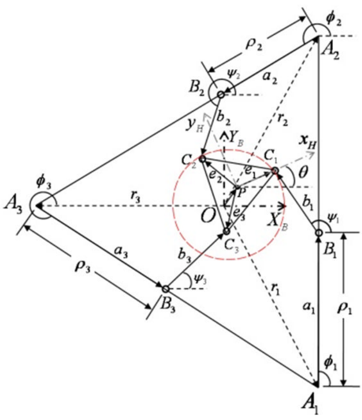

Fig.2：Entire possible workspace (circle)

In this study, Monte Carlo method is applied with following steps[1],

Step1: Consider the entire possible workspace of the manipulator.

Step2: A large number of points (n_{total}) are randomly selected within the circle.

Step3: Each point is tested to determine if it falls within manipulator workspace. This is accomplished by solving the inverse kinematics problem for each leg.

Step4: The number of points that fall within the workspace (n_{in}).

Step5: The workspace area is estimated by the ratio of the points that fall in the workspace to the total number of points selected.

$$W = \frac{n_{in}}{n_{total}} \tag{1}$$

The geometric parameters (*e, r, and b*) of the 3-PRR manipulator are considered as design variables. The bounds of the design variables are given in Table 1.

To avoid intersections between prismatic joint, the prismatic lengths (ρi) are defined.

$$0 \le \rho_i \le \sqrt{3} r \tag{2}$$

Related to the mechanism assembly, the design variables should follow this relation.

$$b + e \ge r / 2 \tag{3}$$

As a result of Table 2, when *e* and *r* has upper and lower bound, respectively, the workspace of the 3-PRR manipulator is maximum value as 0.64. Workspace analysis is achieved by using the general algorithm supposed by Merlet and Gosslin[2]. As the comparison of Fig.3, the workspace was increased greatly after optimization.

Design Variables	*e*	*r*	*b*
Lower Bound [mm]	6	59	35
Upper Bound [mm]	8	61	45

Table 1: Bounds of the design variables

Objective	Design Variables		
W(workspace)	*e*	*r*	*b*
0.64	8	59	41

Table 2: Workspace optimal solutions (n_{total}=1764)

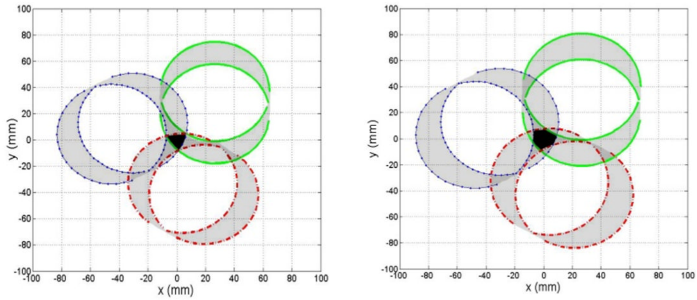

Fig.3: Simulation of workspace (black area). The moving regions of three PRR legs are shown in the Gray parts. (a).Workspace of the PPM before optimization (W=0.37). (b).Optimal workspace of the PPM (W=0.64).

4 Optimization of Dexterity for 3-PRR PPM

The objective of the dexterity optimization is to determine the values of the manipulator design variables that result in the best dexterity. A global condition index (GCI), η, which considers the condition number of the Jacobian over the entire workspace, is defined for the manipulator as:

$$\eta = \int_W \frac{1}{\lambda} dW \qquad 0 < \eta < 1 \tag{4}$$

where λ is the condition number of the Jacobian at a given position in the workspace and W is the manipulator workspace. When η is closer to 1, the manipulator has better dexterity.

In this paper, Monte Carlo method is employed and is outlined as follows:

Steps 1-3 are same as steps 1-3 for Monte Carlo Method.

Step 4: The condition index sum, S, which is the sum of the reciprocal of the condition number of each point that falls within the workspace.

Step 5: The global condition index, η, is determined by and the condition index sum and then dividing by the number of the points within the workspace.

$$\eta = \frac{S}{n_{in}} \qquad 0 < \eta < 1 \tag{5}$$

Table 3 shows the result of dexterity optimization. When e and r has lower bound, and b has upper bound, respectively, the GCI of the proposed 3-PRR manipulator is maximum value as 0.138.

Objective	Design Variables		
GCI	e	r	b
0.138	6	59	45

Table 3: Dexterity optimal solution

Objective		Design Variables		
W	GCI	e	r	b
0.615	0.120	6	59	41

Table 4: Multi-objective optimization result

5 Multi-objective Optimization for 3-PRR PPM

Multi-objective optimization is to find the optimum design parameters of a 3 PRR PPM in order to maximize its workspace and dexterity subject to the same design constraints [3]. This problem can be stated as:

$$\begin{aligned} &\text{maximize} && f_1(x) = W \\ &\text{maximize} && f_2(x) = \eta \\ &\text{over} && x = [e \quad r \quad b]^T \\ &\text{subject to:} && g1 : b + e \geq r/2 \\ & && g2 : 0 < \rho_i < \sqrt{3}r \\ & && x_{lb} \leq x \leq x_{ub} \end{aligned}$$

The optimization problem is solved by MATLAB Optimization Toolbox. The best solution is shown in Table 4. When *e* and *r* has lower bound, the manipulator has a large workspace with high dexterity.

6 Conclusion

In this paper, workspace and dexterity optimization was achieved based on Monte Carlo Method and Global Condition Index, respectively. Furthermore, multi-objective optimization design was carried out to find the optimal variables with comparatively high dexterity and a large workspace. Based on the optimization result, trajectory path of this mechanism will be designed in the future.

7 Acknowledgment

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