Phase-shifting algorithms for the interferometric measurement of form error, diameter variation and refractive index inhomogeneity of fused silica spheres

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Abstract

High-precision spheres have many applications in precision engineering and metrology, such as the calibration of transmission spheres for interferometry, density standards for mass metrology, and spherical gyroscope rotors. For these applications it is important to know the form error of the sphere. Optical interferometry can be applied to measure the form errors of opaque spheres with high spatial point density. Interferometric measurements of transparent spheres, however, are difficult due to the interference of light reflected by the "back surface" of the sphere. Fourier transform phase-shifting interferometry using wavelength tuning can, in principle, separate the fringe patterns originating from the multiple cavities. However, commercially available interferometers require a specific ratio of cavity lengths, which is difficult to realize for sphere measurements.

In this paper we propose the application of phase-shifting algorithms (PSAs) based on characteristic polynomial (CP) theory to measure form error, diameter variation and refractive index inhomogeneity of transparent spheres in a four-surface interference configuration. Computer simulations were used to calculate the intensity variation at a pixel when shifting the frequency of the laser in equal steps. CP theory was used to optimize algorithms to estimate phase maps of three cavities: the full cavity, the cavity formed by the surfaces of the fused silica sphere, and the empty cavity. The phase maps were then combined to obtain the diameter variation of the sphere and its refractive index inhomogeneity. The performance of the algorithms was evaluated by a computer simulation.

1 Characteristic polymonial (CP) theory

The description of phase-shifting algorithms by CPs, introduced by Surrel[1], provides a systematic approach to analyze and optimize an algorithm's frequency response. When the intensity of the k-th phase-shifted interferogram is denoted as I_k (k=0, 1,2,…, M-1), the phase can be calculated according to

$$\phi = \tan^{-1}\left[\sum_{k=0}^{M-1} b_k I_k / \sum_{k=0}^{M-1} a_k I_k\right],$$

where the constants a_k and b_k are the real and imaginary parts of the coefficients of the CP, $P(x)$:

$$P(x) = \sum_{k=0}^{M-1} (a_k + ib_k) x^k$$

By properly choosing the roots of the CP, one can design a phase-shifting algorithm (PSA) that is insensitive to specific harmonics.

2 Four-surface Fizeau interferometer

To measure the form error, diameter variation, and refractive index inhomogeneity of a transparent sphere, we consider a frequency-shifting Fizeau interferometer with a confocal four-surface configuration (Figure 1).

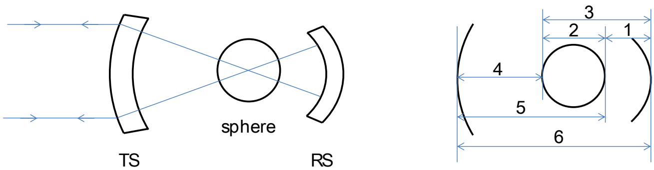

Figure 1: Confocal four-surface configuration of the interferometer. TS: transmission sphere, RS: reflection sphere. The right figure shows the six fundamental cavities.

3 Design of PSAs

3.1 Simulation of the interference signal

The intensity of the overall interferogram was calculated for each stepped frequency. The required cavity lengths were determined using the specification of the spheres available at the National Institute of Standards and Technology (NIST). Figure 2 shows the simulated intensity modulation of the full cavity and its Fourier power spectrum. The power spectrum clearly shows the optical path difference (OPD) ratios of the 6 fundamental cavities. Similar simulations were performed for the empty cavity – the cavity without the sphere.

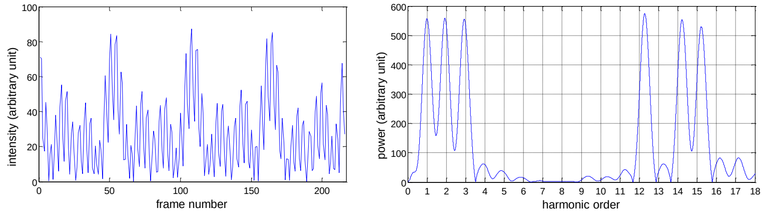

Figure 2: Intensity modulation of the full cavity and its Fourier power spectrum.

3.2 PSAs designed with CP theory

Using the OPD ratios of the cavities, a PSA sensitive to the cavity formed by the front and back surfaces of the sphere, and a PSA sensitive to the full cavity, were designed using the CP theory. Each algorithm suppresses all other basic harmonics of the spectrum. Then, a PSA for the empty cavity was designed. The constants a_k and b_k of the PSA for the full cavity, and the power of the transfer function, are depicted in figure 3.

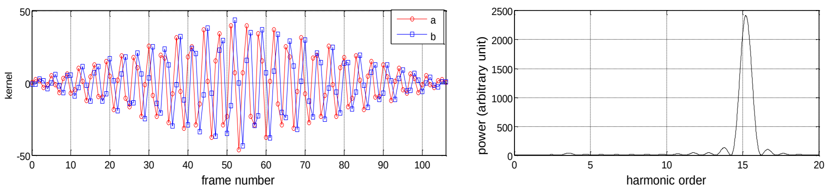

Figure 3: Constants a_k and b_k of the PSA for the full cavity, and the transfer function.

4 DV, RIV and FE calculation

The diameter variation (DV), the refractive index variation (RIV), and the form eror (FE) of the sphere can be calculated as:

$$DV = (\phi_{EC} + \phi_2 - \phi_6) \times \lambda / (4\pi),$$

$$RIV = [\phi_2 - n \times (\phi_{EC} + \phi_2 - \phi_6)] \times \lambda / (4\pi D)$$

$$FE_1 = -\lambda / (4\pi) \times \phi_4 - FE_{TS} \quad \text{(front surface)}$$

$$FE_2 = DV - FE_1 = \lambda / (4\pi) \times (\phi_{EC} - \phi_1) + FE_{TS1} \quad \text{(back surface)}$$

where, ϕ_{EC} and ϕ_i (i=1,2,4,6) denote the phases obtained from the empty cavity and the i-th fundamental cavity (see figure 1), respectively, λ is the wavelength of light, n is the average refractive index of the sphere, D is the average diameter of the sphere, and FE_{TS} is the form error of the reference transmission sphere. FE_{TS} can, for example, be obtained by performing the random ball test [2].

5 Verification of the PSAs by simulation

To verify the PSAs, we generated an arbitrary numeric model for the surface profile and refractive index inhomogeneity of the fused silica sphere. In figure 4, the DV and RIV obtained with the designed algorithms are compared with their nominal values.

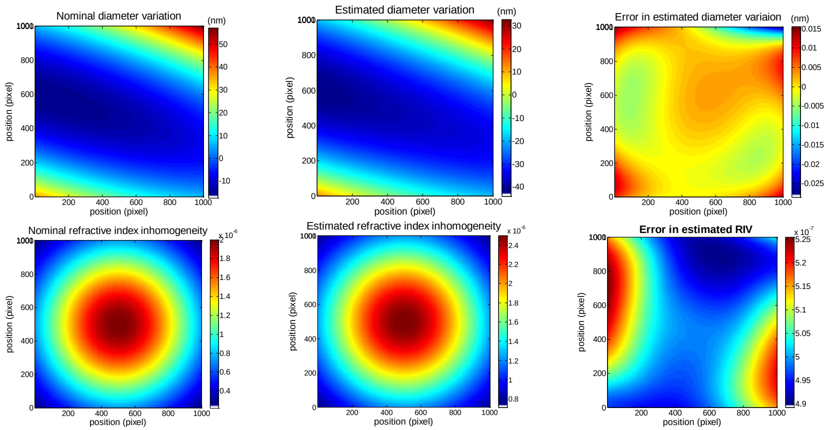

Figure 4: Comparison of the simulated results with the target values.

6 Conclusion

We have designed PSAs which may enable us to measure the DV, RIV, and FE of a transparent sphere. Each PSA is designed to extract the phase information of a specific cavity in a four-surface confocal setup. The phase maps obtained are then combined with the phase map of the empty cavity to estimate the geometric errors of the sphere and the variation in the refractive index for rays through the sphere center.

References:

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