

# A novel stitching algorithm for the measurement of aspheric surfaces

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## Abstract

This paper describes a new stitch algorithm which connects plural partial measurements and combines them into a whole shape. Compared to the conventional stitch method, a significant benefit is that there is no need to determine the overlap region and no need to interpolate three-dimensional point data. These are essential conditions in order to apply the method to a free-form measurement machine whose measured data are sparse and three-dimensional points. The performance of the algorithm is checked by simulation.

## 1 Introduction

For mass producing large and severely aspherical optics, we have been developing a free-form measurement machine utilizing a probe [1] and a novel stitch algorithm. Stitching is currently a popular technique and has already been implemented in some measurement devices [2], but the approach will completely change in the case of a free-form measurement machine. The main cause of the change is that the measurement data is sparse, and is a set of non-aligned three-dimensional points, as described below. The following symbols are used.

$[{}^i x_k, {}^i y_k, {}^i z_k]^T$  : the k-th point of the i-th partial measurement (1)

$z = g(x, y, \mathbf{q})$  : the shape of systematic errors including parameter  $\mathbf{q}$ .

$T(\mathbf{p}_i)$  : the coordinate transfer matrix from the i-th partial measurement to global coordinates. Parameter  $\mathbf{p}_i$  is called the setting error. Generally, the setting error consists of six components, that is, three-dimensional positions and orientations.

The k-th measured point is transferred to global coordinates as

$$[{}^i X_k, {}^i Y_k, {}^i Z_k]^T = T(\mathbf{p}_i)[{}^i x_k, {}^i y_k, {}^i z_k - g({}^i x_k, {}^i y_k, \mathbf{q})]^T \quad (2)$$

These measurement points can be treated as a continuous surface by interpolation.

The interpolating functions corresponding to equations (1) and (2) are denoted as

$z = f_i(x, y)$  : the  $i$ -th partial measurement represented in its local coordinates

$z = F_i(x, y, \mathbf{p}_i, \mathbf{q})$  : the  $i$ -th partial measurement represented in global coordinates

Using these symbols, the mismatch in the overlap region can be written as:

$$\sum_{i \neq j} \sum_{k=\text{overlap}} \left( {}^i Z_k(\mathbf{p}_i, \mathbf{q}) - F_j({}^i X_k, {}^i Y_k, \mathbf{p}_j, \mathbf{q}) \right)^2 \quad (3)$$

The conventional stitch method is based on minimizing the above cost function by adjusting parameters  $\mathbf{p}_i$  and  $\mathbf{q}$  (Fig. 1a). In the case of an interferometer, measured data is small deviation between the workpiece and a reference surface, and can be considered almost as a plane. On the other hand, in the case of a free-form measurement machine, the data is sparse three-dimensional points and small changes in the overlap region lead to significant effects on the whole shape (Fig. 1b). Therefore, it is necessary to determine the overlap region precisely. However, the error in interpolation is large because the points are sparse, and so the conventional stitch method cannot easily calculate the whole shape with practical accuracy.

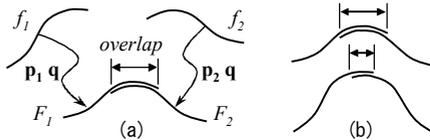


Fig.1 Conventional stitch algorithm

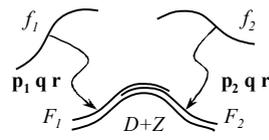
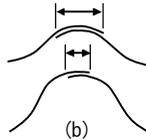


Fig.2 Proposed algorithm

## 2 Stitch algorithm

This problem can be improved by considering the characteristics of the workpiece, namely the aspherical optics. Assume the following:

A1: The design shape of the workpiece is aspherical.

A2: The error shape can be separated into low- and high-frequency components.

With assumption A1, parameter  $\mathbf{p}_i$  can be calculated by fitting to the aspherical design shape. Moreover, it is advantageous because the larger asphericity results in a smaller calculation error in  $\mathbf{p}_i$ . However, this approach alone does not work well since  $\mathbf{p}_i$  is influenced by the shape error of the workpiece. Thus, we simultaneously estimate the shape error, particularly the low-frequency component that affects parameter  $\mathbf{p}_i$ . Embodying assumption A2, the low-frequency component is a Zernike

polynomial defined in the entire region. This component represents an approximated error shape of the workpiece. The new cost function is:

$$\sum_i \sum_k \left( Z_k(\mathbf{p}_i, \mathbf{q}) - D(X_k, Y_k) - Z(X_k, Y_k, \mathbf{r}) \right)^2 \quad (4)$$

where,  $D(x,y)$  is the design shape and  $Z(x,y,r)$  is the Zernike polynomial with coefficient of  $\mathbf{r}$ . The proposed method is to minimize the above cost function by adjusting the parameters  $\mathbf{p}$ ,  $\mathbf{q}$  and  $\mathbf{r}$  (Fig.2). There is no need to interpolate and no need to determine the overlap region. The total stitch algorithm has three steps (Fig.3). Step 1 eliminates harmful high-frequency components by using normal curve fitting technique; Step 2 minimizes the new cost function; and Step 3 adds back the high-frequency component, connects the residual error shape and finally adds back the approximated error shape.

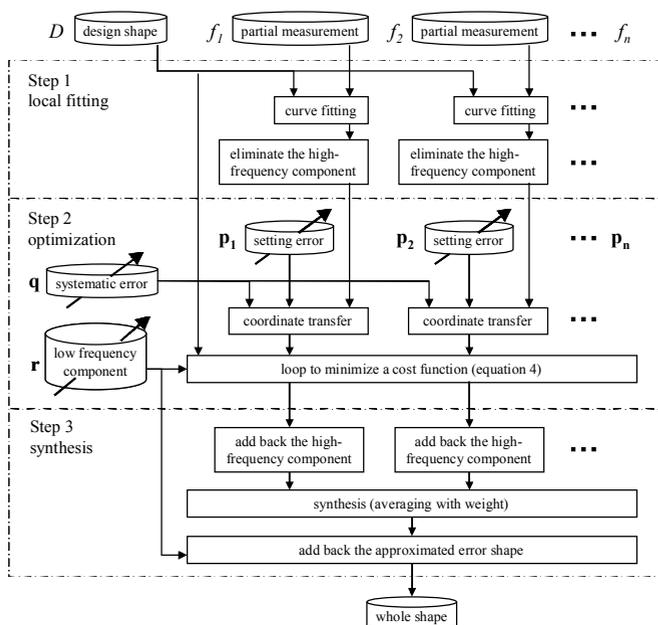


Fig. 3 Flow chart of the proposed stitch algorithm

### 3 Simulation

The validity of this new algorithm is checked by simulation. Figure 5 shows the conditions of the simulation. The target workpiece is a convex hyperboloid surface,

full aperture is 1200 mm and the maximum asphericity is 3.2  $\mu\text{m}$ . The whole area is divided into three fan-shaped partial measurement areas. Position error, orientation error and shape error are intentionally mixed. The shape error includes an axially symmetrical component and a four-fold symmetrical component of 2  $\mu\text{m}$ . The shape error also includes a step-like error shape. The total amount of shape error is 1383 nmRMS. Figure 6 shows the calculation results. The calculation error ( $C=A-B$ ) is 31 nmRMS, which is sufficiently small compared with 1383 nm.

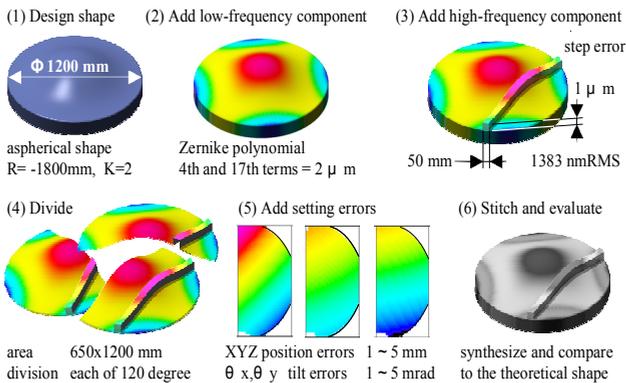


Fig. 5 Simulation procedure and conditions

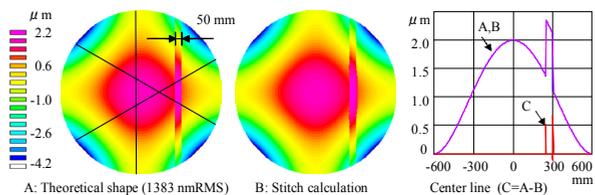


Fig. 6 Stitch result of a test shape including a step-like defect

#### 4 Conclusion

We proposed a novel stitch algorithm which is suitable for free-form measurement machines. A primitive simulation check showed good performance.

#### References:

- [1] M. Negishi et al., "A Double-Sided Contour Measurement Machine", Proc. 6th EUSPEN, pp.273-276(2006)
- [2] J. Fleig et al., "An automated subaperture stitching interferometer workstation for spherical and aspherical surfaces", SPIE 5188, pp.296-307(2003)