Design of Flexure-based Precision Transmission Mechanisms using Screw Theory

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Abstract

This paper enables the synthesis of flexure-based transmission mechanisms that possess multiple decoupled inputs and outputs of any type (e.g. rotations, translations, and/or screw motions), which are linked by designer-specified transmission ratios. A comprehensive library of geometric shapes is utilized from which every feasible concept that possesses the desired transmission characteristics may be rapidly conceptualized and compared before an optimal concept is selected. These geometric shapes represent the mathematics of screw theory and uniquely link a body's desired motions to the flexible constraints that enable those motions. This paper is significant to the design of nano-positioners, motion stages, and optical mounts. Recently, these principles have been applied to the design of transmission mechanisms that constitute the microstructure of new materials with extraordinary properties (e.g. zero/negative thermal expansion coefficients and Poisson's ratios). A hand-actuated microscopy stage was designed, fabricated, and tested to demonstrate the utility of this theory.

1 Introduction

In this paper we introduce the theory that may be used to optimally synthesize complex flexure-based transmission mechanisms. These mechanisms are important to precision engineering because of their ability to amplify or attenuate an input displacement or load on a structure in a repeatable way. As such, they may be used as low-cost solutions for *(i)* increasing the resolution of a system's actuators, *(ii)* improving the sensitivity of a systems sensors, and *(iii)* transforming the nature of a system's inputs (e.g. from rotations to translations). Two sets of geometric shapes have been derived [1] that enable the synthesis of such mechanisms. One set represents the degrees of freedom (DOFs) a system possesses and the other set

represents the constraints that enable those DOFs. Consider, for instance, the flexure shown Fig. 1A. This system possesses a single screw DOF shown as a line along which the stage may translate as it rotates according to the screw's pitch, *p*, shown in Fig. 1B. The screw line is uniquely linked to another shape of circular hyperboloids shown in Fig. 1C from which constraints may be selected that enable the screw motion. The axes of the constraints that lie on the surfaces of these hyperboloids all satisfy the screw line according to

$$p = d \cdot \tan\varphi \,, \tag{1}$$

where d and φ are defined in Fig. 1D. Note from Fig. 1E and Fig. 1F that the constraints of the flexure system shown in Fig. 1A all belong to the shape of Fig. 1C.

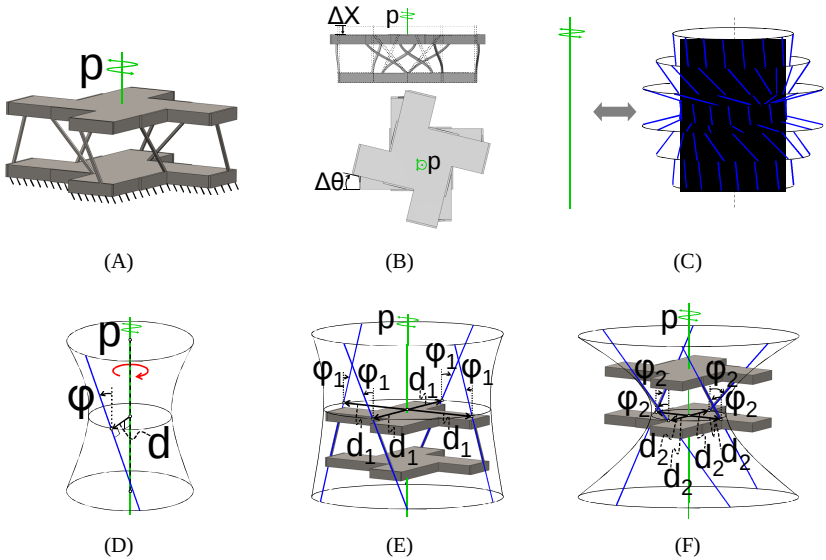


Figure 1: Using geometric shapes to synthesize flexure systems

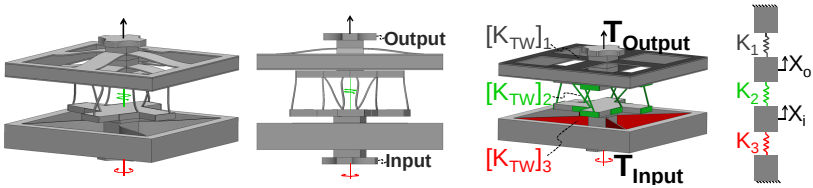
2 Principles of Flexure-based Transmission Synthesis

Other flexure systems that are designed using geometric shapes may be attached to the systems that possess screw DOFs to transform their screw motions into desired transmissions. Suppose, for instance, a system that possessed a single translational DOF were attached to the top plate of the system from Fig. 1A, and a system that possessed a single rotational DOF were attached to the bottom plate as shown in Fig. 2A. If the constraints were ideal (i.e., infinitely stiff along their axes compared to

other directions), the output stage would translate as the input stage rotated as shown in Fig. 2B with a transmission ratio equal to the pitch of the screw from Fig. 1A. The actual transmission ratio is always smaller than this pitch and may be found using

$$\mathbf{T}_{\mathbf{Output}} = [K_{TW}]_1^{-1}\left([K_{TW}]_1^{-1} + [K_{TW}]_2^{-1}\right)^{-1}\mathbf{T}_{\mathbf{Input}}\,, \tag{2}$$

where $[K_{TW}]_1$, $[K_{TW}]_2$, and $[K_{TW}]_3$ are the *6x6* twist-wrench stiffness matrices of each flexure module in the system and $\mathbf{T}_{\mathbf{Output}}$ and $\mathbf{T}_{\mathbf{Input}}$ are the *6x1* twist vectors that describe the input and output displacements as shown in Fig. 2C. See Hopkins [2] to learn how to construct these matrices and vectors for any flexure and displacement. Note from Eq. (2) and from the analogue system shown in Fig. 2D that the twist-wrench stiffness matrices obey the same rules for calculating transmission ratios as linear springs with scalar stiffness values.

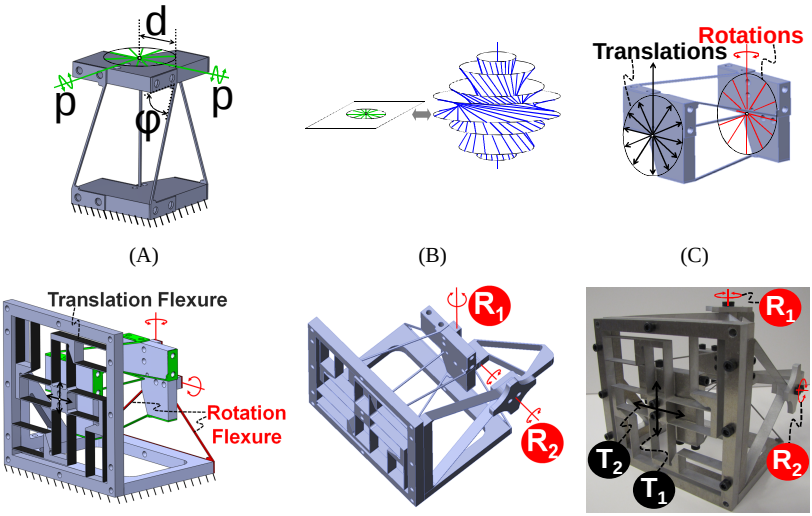


(A) (B) (C) (D)

Figure 2: Synthesis of a flexure-based transmission mechanism using screw theory

3 Synthesizing Multiple Decoupled Input Transmission Mechanisms

The principles of this paper may be applied to the synthesis of transmission mechanisms with multiple decoupled inputs. Consider the flexure system from Fig. 3A that possesses two intersecting screw DOFs represented by a disk of screws. These screws share a common pitch, *p*, that may be found using Eq. (1) for the parameters labeled in Fig. 3A. The system's constraints lie on the surfaces of nested circular hyperboloids shown in Fig. 3B. A transmission ratio less than the pitch of these screws could be achieved by attaching flexure systems that would constrain the top plate of the system to rotate about the axes of the screw lines and the bottom plate to translate in the direction of the screw lines as shown in Fig. 3C. Examples of such flexure attachments are shown in Fig. 3D. In order to decouple the rotational input *R1* from *R2*, the flexures shown in Fig. 3E may also be attached. Similar flexures could be attached ontop of the system to decouple *R2* from *R1*. The resulting 2 DOF flexure-based transmission mechanism is shown in Fig. 3F.



(A) (B) (C)

(D) (E) (F)

Figure 3: Synthesis of a multi-input decoupled flexure-based transmission mechanism

Conclusion

We have provided a synopsis of a method wherein designers may use geometric shapes to synthesize flexure-based transmission mechanisms that possess multiple inputs that link to decoupled outputs of various types with designer-specified transmission ratios. A flexure-based microscopy stage with two decoupled rotational inputs that link to two translational outputs was fabricated and tested. This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344. LLNL-CONF-469875.

References:

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