Optimal Strategy for Producing a High Quality Relay in a Machining-assembly Production System Applied a Corrective Assembly Approach

T. Iyama¹, M. Mizuno¹, N. Yoshihara¹, N. Nisikawa¹
¹Iwate University, Japan

iyama@iwate-u.ac.jp

Abstract
In this paper, we propose an optimal strategy to maximize a production rate of high quality relay that satisfies a predetermined assembly tolerance in a relay machining-assembly production system applied a corrective assembly approach. In this strategy, an optimal adjusting machine is selected online according to a selection-probability of each adjusting machine. At first, the strategy is formulated. Next, the optimality of the strategy and effects of some parameters are presented.

1 Introduction
A relay is one of important electric devises, and a higher quality relay has been required. A relay is usually composed of two main parts, and a tight assembly tolerance is required to satisfy the high quality. However, machining errors occur when assembly parts are produced, so a corrective assembly approach¹ is applied to a machining-assembly production system. In the corrective assembly approach, a pair of assembly parts is selected in a production line, and is assembled after one of the parts is reprocessed by a selected adjusting machine. However, as the required assembly tolerance becomes tighter, the production rate of high quality relay drops because a measurement error and a reprocessing error occur in a measuring process and an adjusting process, respectively. These errors cause an erroneous selection of the adjusting machine and an unpredictable assembly error.

In this paper, we propose an optimal strategy to maximize the production rate of high quality relay that satisfies a predetermined assembly tolerance in a relay machining-assembly production system applied the corrective assembly approach. At first, the strategy is formulated. Next, the optimality of the strategy, and effects of a predetermined assembly tolerance and a reprocessing accuracy on the maximum
2 Production system and corrective assembly approach

A relay is generally composed of two main assembly parts; armature (part A) and base (part B). Parts A and B have design dimensions \( L_A \) and \( L_B \), respectively. An assembly dimension \( L_B - L_A \) determines the quality of relay, and a predetermined assembly tolerance is set. A relay production system is composed of machining stages \( S_A \) and \( S_B \), measuring stages \( S_{MA} \) and \( S_{MB} \), reprocessing stage \( S_C \), and assembly stage \( S_D \). In stage \( S_C \), \( NK \) adjusting machines with a different adjustment size are installed to adjust the part A so as to satisfy the predetermined assembly tolerance.

In the system, machining errors \( \Delta W_A \) and \( \Delta W_B \) occur in stages \( S_A \) and \( S_B \) in producing parts A and B, and measurement errors \( \Delta M_A \) and \( \Delta M_B \) occur in stages \( S_{MA} \) and \( S_{MB} \) in measuring the machining errors of parts A and B, respectively. As a result, when a pair of parts A and B is selected in a production line to be assembled, a true assembly error \( \Delta Y = (L_B + \Delta W_B) - (L_A + \Delta W_A) - (L_B - L_A) = \Delta W_B - \Delta W_A \), and an estimated assembly error \( \Delta Y^* = (L_B + \Delta W_B + \Delta M_B) - (L_A + \Delta W_A + \Delta M_A) - (L_B - L_A) = (\Delta W_B - \Delta W_A) + (\Delta M_B - \Delta M_A) \). Furthermore, a reprocessing error occurs in stage \( S_C \) in adjusting the part A. Consequently, when an adjusting machine \( M_i \) (\( i = 1, 2, \ldots, NK \)) is selected, a true reprocessing assembly error \( \Delta Y^+ \) after reprocessing is given by Eq. (1).

\[
\Delta Y^+ = (L_B + \Delta W_B + \Delta M_B) - (L_A + \Delta W_A + E_i + \Delta E_i) - (L_B - L_A) = \Delta W_B - (\Delta W_A + E_i + \Delta E_i), \tag{1}
\]

where \( E_i \) and \( \Delta E_i \) are the adjustment size and a reprocessing error of \( M_i \). If adjusting is not necessary, \( E_i = \Delta E_i = 0 \). In this process, the true assembly error \( \Delta Y \) is unknown, so the adjusting machine is selected using the estimated assembly error \( \Delta Y^* \), and an erroneous selection of the adjusting machine occurs.

3 Adjusting machine selection method

In the corrective assembly approach, to maximize the production rate of the relay which satisfies the predetermined assembly tolerance, the optimal adjusting machine which maximizes a selection-probability that \( \Delta Y^+ \) is within the predetermined assembly tolerance must be selected. In this section, we formulate the selection-probability \( PE_i(t) \) of adjusting machine \( M_i \) in the case where the estimated assembly error \( \Delta Y^* = t \), and the reprocessing error \( \Delta E_i = y \).
The p.d.f. (probability density function) \( f_w(s) \) of the true assembly error \( \Delta Y = \Delta W_B - \Delta W_A = s \), and the p.d.f. \( f_m(x) \) of the measurement assembly error \( \Delta Y^* = \Delta M_B - \Delta M_A = x \) for the product assembled by a pair of parts A and B are given by Eq. (2).

\[
\begin{align*}
\int_{-\infty}^{\infty} f_{WA}(\delta) f_{WB}(s + \delta) d\delta, \\
\int_{-\infty}^{\infty} f_{MA}(\delta) f_{MB}(x + \delta) d\delta,
\end{align*}
\]

where \( f_{WA}(\cdot) \) and \( f_{WB}(\cdot) \) are the p.d.f.s of machining error of parts A and B, and \( f_{MA}(\cdot) \) and \( f_{MB}(\cdot) \) are the p.d.f.s of measurement error of parts A and B, respectively. When the estimated assembly error \( \Delta Y^* = (\Delta W_B - \Delta W_A) + (\Delta M_B - \Delta M_A) = s + x = t \), and the adjusting machine \( M_i \) is selected, Eq. (3) must be true for the true reprocessing assembly error \( \Delta Y^* = t - x - E_i^{-} - y \) to satisfy the predetermined assembly tolerance \(|K_\delta|\).

\[
-K_\delta \leq t - x - E_i^{-} - y \leq K_\delta.
\]

In this case, \( y \) is any value, so \( PE_i(t) \) is given by Eq. (4).

\[
PE_i(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_M(x) f_w(t - x) f_{Ei}(y) dy dx,
\]

where \( f_{Ei}(\cdot) \) is the p.d.f. of reprocessing error. Consequently, the adjusting machine which maximizes the selection-probability \( PE_i(t) \) is selected as the optimal machine. The maximum production rate \( R_g^* \) yielded in this strategy is given by Eq. (5).

\[
R_g^* = \max_{i=1, \ldots, NK} \int_{-\infty}^{\infty} PE_i(t) dt.
\]

### 4 The optimality of the proposed strategy

To present the optimality of the proposed strategy, we compare the maximum production rate \( R_g^* \) obtained by Eq. (5) with the production rate \( R_g \) obtained by various estimated assembly error ranges \([I_i^*, J_i^*]\) \((i=1, 2, \ldots, NK)\) in which a pair of parts A and B selects the adjusting machine \( M_i \). The system parameters are \( NK=3 \), \( E_1=2|K_\delta|, E_2=0, E_3=2|K_\delta|[\mu m], -I_1^* = J_3^* = \infty, \) and \( -I_2^* = J_2^* = (J_3^*), \) and the other parameters are shown in Table 1. The results for \( |K_\delta|=10,20[\mu m] \) and \( 3\sigma_{Ei}=0,2.5,5,7.5,10[\mu m] \) are shown in Fig.1, where ‘x’ in Fig.1 denotes \( R_g^* \). Table 2 shows \( R_g^* \), the maximum production rate \( R_{gmax} \) in \( R_g \), and the estimated assembly error range \([I_i^*, J_i^*]\) which yields \( R_{gmax} \). From Fig. 1 and Table 2, it is presented that \( R_g^* \) gives the maximum production rate in all \(|K_\delta|\) and \( 3\sigma_{Ei} \). This means that the optimal adjusting machine is selected using the proposed strategy to maximize the production rate. Furthermore, Table 2 presents that the reprocessing error decreases
$R_g^*$ as $|K_0|$ becomes tighter, but has little effect on $[I^*, J^*]$ which yields $R_{g_{max}}$ when $|K_0|$ is constant.

Table 1: Basic system parameters.

<table>
<thead>
<tr>
<th></th>
<th>Normal distribution (mean: $\mu_{WA}=\mu_{WB}=0$ [μm], standard deviation: $\sigma_{WA}, \sigma_{WB}$ [μm])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distribution of $\Delta W_A, \Delta W_B$</td>
<td>Normal distribution (mean: $\mu_{MA}=\mu_{MB}=0$ [μm], standard deviation: $\sigma_{MA}, \sigma_{MB}$ [μm])</td>
</tr>
<tr>
<td>Distribution of $\Delta M_A, \Delta M_B$</td>
<td>Normal distribution (mean: $\mu_{E_i}=0$ [μm], standard deviation: $\sigma_{E_i}$ [μm])</td>
</tr>
<tr>
<td>Distribution of $\Delta E_i (i=1,2,3)$</td>
<td>Normal distribution (mean: $\mu_{E_i}=0$ [μm], standard deviation: $\sigma_{E_i}$ [μm])</td>
</tr>
</tbody>
</table>

Machining accuracy of parts A, B $\ (3\sigma_{WA}, 3\sigma_{WB}) =$ (30,15) [μm]

Measurement accuracy of parts A, B $\ (3\sigma_{MA}, 3\sigma_{MB}) =$ (10,10) [μm]

Reprocessing accuracy of $M_i (i=1,2,3)$ $\ 3\sigma_{E_i}$ [μm]

5 Conclusions

1) The strategy which yields the maximum production rate is proposed, and the optimality of the strategy is presented.

2) The reprocessing error decreases the maximum production rate as the assembly tolerance becomes tighter.

Fig. 1: Optimality of the proposed strategy

| $|K_0|$ | $3\sigma_{E_i}$ | $R_g^*$ | $R_{g_{max}}$ | $J^*_1=I^*_2$ | $J^*_2=I^*_3$ |
|---|---|---|---|---|---|
| 10 | 0 | 0.7366 | 0.7366 | -12 | 12 |
| 2.5 | 0.7346 | 0.7346 | -12 | 12 |
| 5 | 0.7292 | 0.7292 | -12 | 12 |
| 7.5 | 0.7208 | 0.7208 | -12 | 12 |
| 10 | 0.7100 | 0.7100 | -12 | 12 |

20 | 0 | 0.8905 | 0.8905 | -24 | 24 |
| 2.5 | 0.8896 | 0.8896 | -24 | 24 |
| 5 | 0.8884 | 0.8884 | -24 | 24 |
| 7.5 | 0.8865 | 0.8865 | -24 | 24 |
| 10 | 0.8843 | 0.8843 | -24 | 24 |

References: