# Prediction of 5-DOF Motion Errors of Hydrostatic Bearing Tables and Its Experimental Verification

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## 1 Introduction

In precision feed tables such as aerostatic or hydrostatic tables, 5-DOF motion errors are largely affected by the profile errors of guide rails. To get the target precision of the feed tables, final finishing process by the skilled workers is normally required and it is time consuming and iterative process. If it is possible to predict the motion errors of the table from the rail profiles, the allowable tolerance of the rail profile can be defined in a design step and also the time for finishing process can be reduced drastically. In a normal hydrostatic bearing table, several bearing pads with same size are placed in a table structure. So, if it is possible to analyze the characteristics of single bearing pad and the spatial arrangement of the bearing pads are known, transfer function method can be effectively utilized in the analysis of motion error [1]. In this paper, we introduce a new analytical model and algorithm based on the transfer function method for predicting 5-DOF motion errors of a hydrostatic bearing table. To verify the validity of the proposed algorithm, rail profile errors of real hydrostatic bearing table are measured and the estimated motion errors are compared with real motion errors of hydrostatic bearing table.

## 2 Analytical Model of 5-DOF Motion Errors

Figure 1 shows the behaviour of single hydrostatic bearing pad moving along the rail with sinusoidal profile. Bearing follows the period of rail profile but the magnitude is changed by the amount of film reaction force which is determined by the spatial frequency and magnitude of the rail profile error. Transfer function is defined as the variation of the film force in a pad corresponding to the magnitude of the spatial frequency component of the rail profile error when the bearing pad moves straight along one period of the rail profile.

$$K(\omega) = \frac{f_e(\omega)}{e(\omega)} \tag{1}$$

Transfer function can be obtained by calculating the variation of load characteristics corresponding to the specific spatial frequency of rail profile by use of FEM analysis.

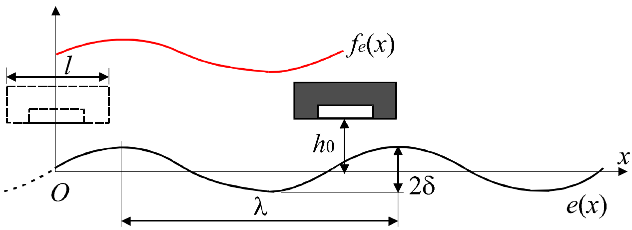


Figure 1: Response of film reaction force to sinusoidal profile of rail

Figure 2 shows the analytical model of 5-DOF motion errors of hydrostatic bearing table. 5-DOF motion errors of the table center is defined as $\delta_y$, $\delta_z$, $\theta_x$, $\theta_y$, $\theta_z$ respectively. Characteristics of bearing pads including stiffness in each direction is assumed to be same.

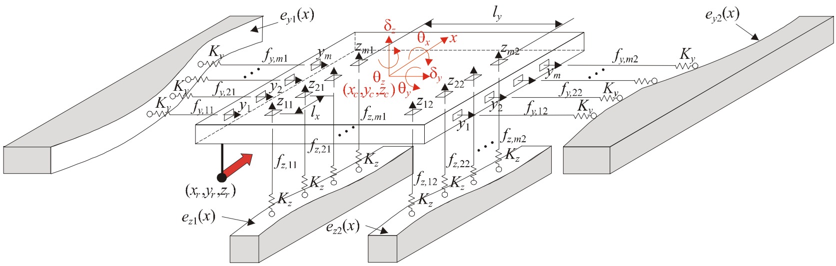

Figure 2: Analytical model of 5-DOF motion errors

Then, the displacement of each bearing pad can be expressed by its geometrical arrangement and 5-DOF motion errors.

$$z_{i,j} = \delta_z + \theta_y X_{vci} - \theta_x Y_{cj} \tag{2}$$

$$y_{i,j} = \delta_y - X_{hci}\theta_z - \theta_x z_c \tag{3}$$

Where $X_{vci}$, $X_{hci}$, $Y_{cj}$ represent the relative distance between the center of bearing pad and the table center. From Eq. (2) and (3), force and moment-equilibrium equation can be applied to the analytical model.

$$\{\delta\}=[A]^{-1}([B]\{F\}+\{M\}) \tag{4}$$

$$\{\delta\}=\{\delta_z \quad \theta_y \quad \theta_x \quad \theta_z \quad \delta_y\}^T$$

$$[A]=\begin{bmatrix} 1-\overline{K}_z & -\overline{K}_z D_x & \overline{K}_z D_y & 0 & 0 \\ -\overline{K}_z D_x & A_{0v}-\overline{K}_z D_x^2 & \overline{K}_z D_x D_y & 0 & 0 \\ -\overline{K}_z D_y & -\overline{K}_z D_x D_y & \overline{K}_z D_y^2+\overline{K}_z D_z^2-B_0-C_K z_c^2 & -\overline{K}_z D_x D_z & C_K z_c+\overline{K}_z D_z \\ 0 & 0 & -\overline{K}_y D_x D_z & \overline{K}_y D_x^2-A_{0h} & -\overline{K}_y D_x \\ 0 & 0 & -z_c-\overline{K}_y D_z & \overline{K}_y D_x & 1-\overline{K}_y \end{bmatrix}$$

$$[B]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & z_c C_K \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \{M\}=\begin{Bmatrix} f_{z,e}/K_A \\ (f_{z,e}D_x+M_{ry})/K_A \\ (f_{z,e}D_y-f_{y,e}D_z)/K_A \\ (f_{y,e}D_x+M_{rz})/K_B \\ f_{y,e}/K_B \end{Bmatrix}$$

$$\{F\}=\begin{Bmatrix} \frac{1}{K_A}\sum_j^{n_v}\sum_i^{m_v} f_{z,ij} \\ \frac{1}{K_A}\sum_j^{n_v}\sum_i^{m_v} f_{z,ij}(X_{vci}+\gamma_{vi}) \\ \frac{1}{K_A}\sum_j^{n_v}\sum_i^{m_v} f_{z,ij}Y_{cj} \\ \frac{1}{K_B}\sum_j^{n_h}\sum_i^{m_h} f_{y,ij}(X_{hci}+\gamma_{hi}) \\ \frac{1}{K_B}\sum_j^{n_h}\sum_i^{m_h} f_{y,ij} \end{Bmatrix}$$

$$A_{0v}=\frac{(m_v^2-1)l_{xv}^2}{12}, \quad A_{0h}=\frac{(m_h^2-1)l_{xh}^2}{12}, \quad B_0=\frac{(n_v^2-1)l_y^2}{12}$$

$$K_A=n_v m_v K_z, \quad K_B=n_h m_h K_y, \quad C_K=K_B/K_A$$

$$\overline{K}_z=K_r/K_A, \overline{K}_y=K_r/K_B$$

Matrix [A] shows the geometrical relation and stiffness. $K_z$ and $K_y$ represent the stiffness of bearing pad in z- and y-direction respectively, while $K_r$ represent the stiffness of feed mechanism such as ball screw. $m_v$ and $m_h$ respresent the number of vertical and horizontal pad in x-direction, while $n_v$ and $n_h$ respresent those in y-direction. $D_x$, $D_y$, $D_z$ is the relative coordinate of feed machanism with respect to the table center. Matrix {F} shows the effect of film reaction force inside the bearing corresponding to the profile error of the rail. $\gamma_{vi}$, $\gamma_{hi}$ is the distance between the moment center and geometrical center in each pad. If the profile error of the rail is given, film reaction force of the specific pad can be calculated by use of the Fourier coefficient of profile error and transfer function.

$$f_{z,ij}(x_i) = \sum_{k=1}^{n} K\left(\frac{2k\pi}{L}\right)\left(a_{k,zj}\cos\frac{2k\pi}{L}x_i + b_{k,zj}\sin\frac{2k\pi}{L}x_i\right) \quad (5)$$
$$x_i = x + X_{ci}$$

,where L is the length of rail. Matrix {M} represent the effect of external force.

## 3 Experimental Results and Discussion

To verify the validity of proposed predition algorithm, comparative experiment was done for real hydrostatic table. Rail profile was measured by use of special straightness measurement device [2] and then the motion errors of the table were calculated by use of the proposed prediction algorithm. Figure 3 shows the results of comparison. Predicted motion errors show good agreement with experiments. Proposed algorithm can also be applied to another types of linear motion tables such as aerostatic or LM bearing tables since the transfer function for such bearings can be obtained with similar procedure.

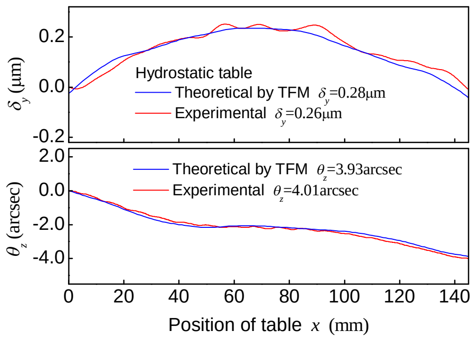


Figure 3: Comparison of predicted motion errors with experiments

**References:**

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[2] C.H. Park, J.H. Chung, S.T. Kim, H. Lee, “Development of a submicron order straightness measuring device,” Journal of KSPE, Vol. 17, No. 5, pp. 124-130, 2000