Ultra-precision Freeform Surface Evaluation Using Genetic Algorithm Optimization

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Abstract

Freeform surfaces have been widely used in industry and daily life for their unique optical properties. To meet the required performances, they are usually needed to be manufactured with extremely high form accuracy. To obtain the form error, one of the main challenges is to match the measured surface data with theoretical model. However, due to the surface complexity, matching results may be trapped at a local minimum. In this proposed paper, a high efficient iteration method was developed to calculate closest point on the designed surface to the measured point and genetic algorithm has been proposed to find out the optimized motion parameter. From the experimental results, it can be seen that the form error caused by the evaluation algorithm can reach nanometric level.

1 Development of form error evaluation model

Measured freeform surface is usually represented by discrete points which can be represented by $M = \{\mathbf{m}_k \in \mathrm{R}^{3\times1} | \ k = 1, 2, \ldots, s\}$.The corresponding point which is the one on designed surface and has the minimum distance to specific measured point as $N = \{\mathbf{n}_k \in \mathrm{R}^{3\times1} | \ k = 1, 2, \ldots, s\}$. Assuming d_k is the distance between the kth measured and its corresponding point, and it can be expressed as:

$$d_k(\boldsymbol{X}) = \left\| \boldsymbol{n}_k - \left(\boldsymbol{R}(\boldsymbol{X}) \boldsymbol{m}_k + \boldsymbol{t}(\boldsymbol{X}) \right) \right\| \tag{1}$$

Where $\boldsymbol{R}$ is the rotation matrix and $\boldsymbol{t}$ is the translation vector and $\boldsymbol{X}$ is the translation and rotation vector: $\boldsymbol{X}= [T_x\ T_y\ T_z\ \theta_x\ \theta_y\ \theta_z]$. The matching process can be generalized as translating and rotating the measured data in workpiece coordinate system and make these two surfaces as close as possible, as illustrated by figure 1.

According to the minimum zone criterion, the matching process can be generalized as an optimization problem with the following objective function [1].

$$\min E(\boldsymbol{X}) = \min\left(\left| \max\left(d_k\right) - \min\left(d_k\right) \right| \right) \qquad k = 1, 2, \ldots, s \tag{2}$$

The optimization problem is to find the translation and rotation parameters to fulfil the objective function.

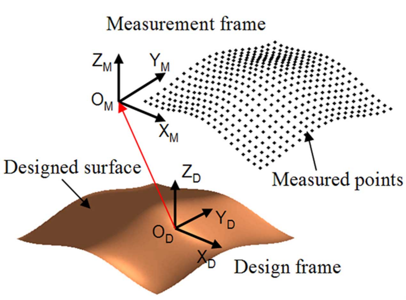

Figure 1: Illustration of freeform surface matching process

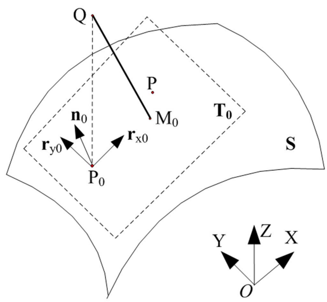

Figure 2: Point to surface distance calculation

An iteration method is adopted for the corresponding point calculation as illustrated by figure 2, where Q is a measured point, P_0 is the initial iteration point, P is the corresponding point, **S** is the designed surface, T_0 is a tangent plane which is constructed by the coordinate of P_0 and its tangential vector $\mathbf{r}_{x0}$ and $\mathbf{r}_{y0}$. Assuming M_0 is the projection point of Q on T_0, its x and y coordinate can be obtained by solving the straight line equation and the tangent plane, for the first iteration, they are x_1 and y_1, respectively. A new point P_1 with the coordinate of $(x_1, y_1, S(x_1, y_1))$ can be considered the start point for the next iteration. For each iteration, the distance between measured point Q and the *l*th iteration point P_l is also calculated. If $||Q P_l| - |Q P_{l-1}||<\varepsilon$, P_l can be considered as the corresponding point of Q, where ε is a given allowed tolerance.

2 Solving the optimization problem using genetic algorithm (GA)

Many methods have been developed such as iterative closet point (ICP) method [2] and Sequential quadratic programming (SQP) method [3], but these methods may not obtain global minimum points. On the other hand, GA is very appropriate for complex non-linear models where location of the global optimum is a difficult task. The typical design of the classical GA can be generalized as the following steps [4]: (1) randomly generate N chromosomes as the initial source population, (2) calculate the fitness of each chromosome of the source population, (3) create the next generation population by chromosome selection, crossover and mutation, (4) replace the source population with the successor population (5) To judge whether the stopping criteria has been met, if not go to step (2). To this freeform surface evaluation problem, the fitness function is the objective function (2), and optimize parameters is ***X***.

3 Experimental studies and discussion

To verify GA based ultra-precision freeform surface evaluation method, sinusoidal grid surface was selected as the designed surface which can be expressed as the following equation.

$$z = A_x \sin\left(\frac{2\pi x}{\lambda_x}\right) + A_y \sin\left(\frac{2\pi y}{\lambda_y}\right) \tag{3}$$

Where, the wavelengths in x and y direction are $\lambda_x = \lambda_y$ =1000μm, and the amplitudes are A_x =A_y=2.5μm. Then a portion of this surface with the area of －1.2mm≤x≤0.6mm, －1.2mm≤y≤0.6mm was selected, then translated and rotated to an arbitrary position to simulate the measured surface. In this presented paper, initial matching is achieved by removing the tilting error of the measured surface using least-squares plane fitting and aligning it with the designed surface through the maximum value points. Initial matching result is illustrated by figure 3 (a), though the measured surface is generally under designed surface, a relatively good registration result has been achieved with the maximum error of 0.196μm and the minimum error of －1.439μm as shows by figure 3 (b). Matlab based GA toolbox was adopted for fine matching optimization solving and the basic parameters settings were as follows: Crossover rate 0.2, Population size 300, maximum iteration step 100，fitness error 10^{-6}μm, Mutation function Gaussian. Other parameters are default values. Figure 3(c) is the fine matching result and figure 3(d) is the surface error. The final matched error E can reach 13.76nm and a high precision matching has been reached.

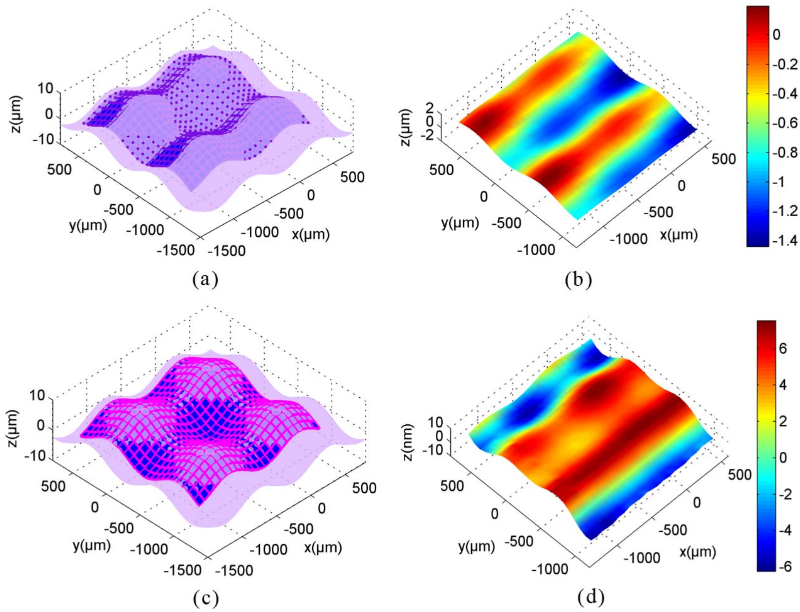

Figure 3: (a) Initial matching result, (b) micrometric initial matching error, (c) fine matching result, (d) nanometric fine matching error

In actual problem solving process, the iteration stopped when the generation is 59. This means at this generation, this variation of the fitness change is less than 10^{-6}µm, such that designed and measured surface has well matched.

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