Error Estimation for the Stitched Profile in Straightness Measurement

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Abstract

Stitching in profile measurement is a technique to expand measurement distance by connecting partly overlapped profiles. We analytically estimated error in the stitched profile based on error propagation rules and evaluated it through experiment. The estimated value agreed fairly well with the experimental value.

1 Introduction

Stitching in profile measurement enables to expand measuring distance by connecting partly overlapped profiles. However, it is necessary to estimate error in the stitched profile and confirm that the error is tolerable in advance, because it can increase error through its extrapolation process. In this paper, we analytically estimated error in the stitched profile based on error propagation rules and evaluated it through experiment.

2 Error estimation

Figure 1 shows stitching using least square approximation lines. $f(x)$ is the profile we aim to measure. $f_i(x)$ is the i-th profile actually measured, where o_i and r_i are its start and end point. Each $f_i(x)$ overlaps with its neighbouring profiles $f_{i-1}(x)$ and $f_{i+1}(x)$, where p_i is starting point of the overlapping range with $f_{i+1}(x)$, and q_i is end of the overlapping range with $f_{i-1}(x)$. Considering that each neighbouring profiles are stitched as their least square approximation lines become identical in their overlapping range, the n-th profile: $f_n(x)$ is expressed by

$$f_n(x) = f_1(x) + \sum_{i=1}^{n-1} \left(\left[y_{i+1}(x) \right]_{o_{i+1}}^{q_{i+1}} - \left[y_i(x) \right]_{p_i}^{r_i} \right), \tag{1}$$

where $\left[y_i(x) \right]_{p_i}^{r_i}$ is the least square approximation line derived from the measurements for $f_i(x)$ with the range from p_i to r_i.

If we expand the definition range for each $f_i(x)$ (i=1 to n) into the total measurement range: [0,l], the first profile: $f_1(x)$ and the n-th profile: $f_n(x)$ can be considered to be the profile obtained without stitching and that obtained through n-times of stitching, respectively. Then equation 1 expresses that the profile obtained from n-times of stitching: $f_n(x)$ can be expressed by the profile obtained without stitching: $f_1(x)$ and difference between the least square approximation lines for each overlapping range. It follows that error propagation for the stitched profile can be estimated from the error for the profile without stitching and those for the least square approximating lines.

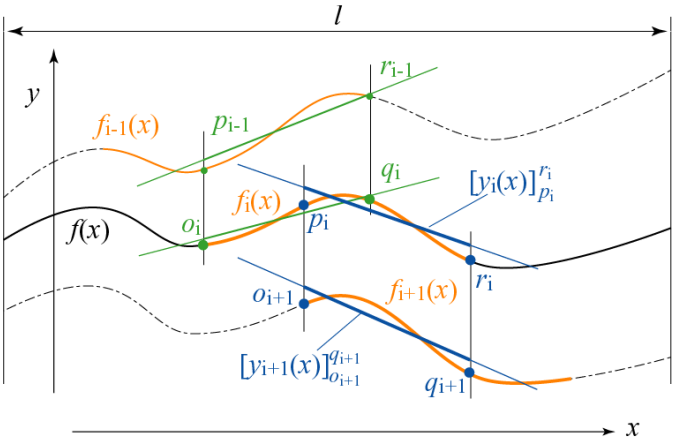

Figure 1: Stitching using least square approximation lines, where l expresses the total measurement distance.

Figure 2 shows analysis model for our error estimation. Each $f_i(x)$ has a unit measurement length: lu and an overlapping length: $k*lu$, where k is an overlapping ratio. Each measurement was done with a sampling interval: s and each overlapping range is equally divided into m by s. Then the least square approximation lines: $[y_i(x)]_{o_i}^{q_i}$ and $[y_i(x)]_{p_i}^{r_i}$ can be expressed as

$$[y_i(x)]_{o_i}^{q_i} = c\sum_{j=o_i}^{q_i}\left[\left\{(m+1)x - \sum_{j=o_i}^{q_i} x_{ij}\right\}x_{ij} + \sum_{j=o_i}^{q_i} x_{ij}^{\ 2} - \sum_{j=o_i}^{q_i} x_{ij}\cdot x\right]y_{ij}, \tag{2}$$

$$[y_i(x)]_{p_i}^{r_i} = c\sum_{j=p_i}^{r_i}\left[\left\{(m+1)x - \sum_{j=p_i}^{r_i} x_{ij}\right\}x_{ij} + \sum_{j=p_i}^{r_i} x_{ij}^{\ 2} - \sum_{j=p_i}^{r_i} x_{ij}\cdot x\right]y_{ij} \tag{3}$$

by using measurements: (x_{ij}, y_{ij}), where $o_i = (i-1)\cdot(1-k)\cdot lu$, $p_i = o_i + \left(\frac{1}{k}-1\right)\cdot m\cdot s$, $q_i = o_i + m\cdot s$, $r_i = o_i + \frac{m\cdot s}{k}$, $c = \frac{12}{s^2\cdot m\cdot(m+1)^2\cdot(m+2)}$, respectively.

On the other hand, equation 1 can be transformed as

$$f_n(x) = f_1(x) - [y_1(x)]_{p_1}^{r_1} + \sum_{i=2}^{n-1}\left([y_i(x)]_{o_i}^{q_i} - [y_i(x)]_{p_i}^{r_i}\right) + [y_n(x)]_{o_n}^{q_n}. \qquad (4)$$

If we consider that there is no error in x_{ij}, which stands for each measurement position, and that error in each y_{ij}, which stands for each measured profile, is random and independent with each other, error propagating to the stitched profile: σ_s can be derived as $\sigma_s = K_e * \sigma_d$. K_e is a coefficient for error propagation which is functions of l, lu, s, and k. σ_d is error in each measurement: y_{ij}. σ_s can be derived analytically by obtaining root mean square of errors in each right side term for equation 4 based on error propagation rules.

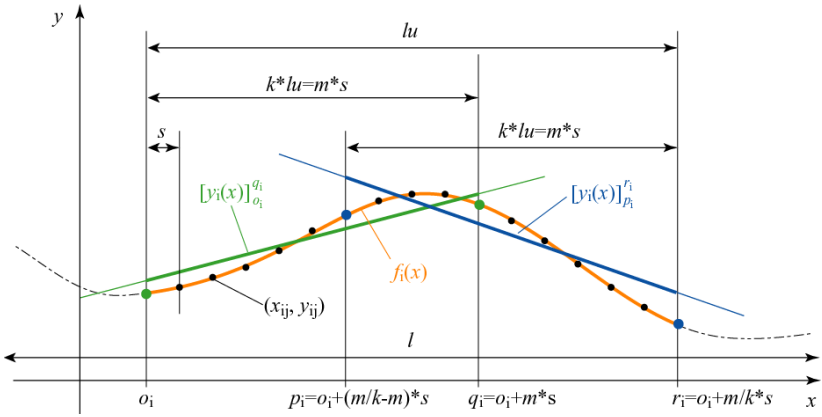

Figure 2: Analysis model for the error propagation. (x_{ij}, y_{ij}) stands for the j-th measurement for the i-th profile.

3 Evaluation for the estimation through experiement

We evaluated our error estimation through experiment. The experimental values were obtained as follows. Each profile to be stitched was divided from the 10-times of repeat measurements of our straightness measurement system.[1] Each measurement has a total measurement length: l of 1400 mm and a sampling interval: s of 4 mm.

Each divided profile has unit measurement lengths: *lu* of 280, 440 and 800 mm and overlapping ratios: *k* from 0.2 to 0.925. The stitched profiles were obtained by connecting profiles divided from different measurements. The standard deviation between the stitched profiles obtained from different sets of divided profile was adopted as the experimental error. Then, ratio of the standard deviation between the original single measurements and the stitched profiles was obtained as an experimental K_e. Figure 3 shows comparison between the estimated and the experimental K_e. The experimental values are for two sets of measurements. As shown in fig. 3, the estimated value agreed fairly well with the experimental value.

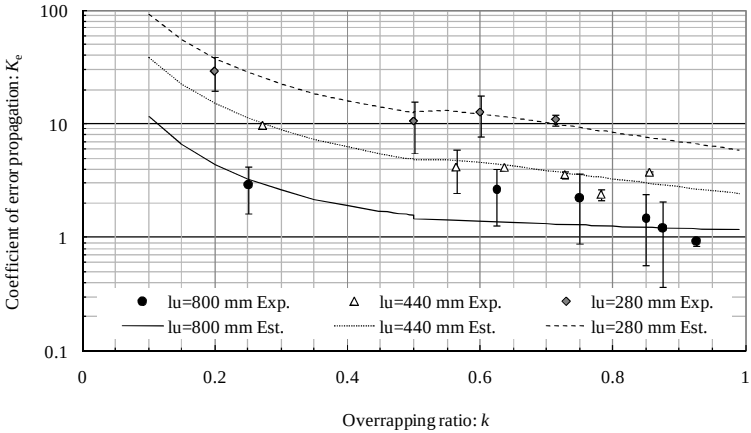

Figure 3: Comparison between the estimated and the experimental K_e. Error bars show standard deviations for each experimental value.

4 Conclusion

We estimated error in the stitched profiles based on error propagation rules and evaluated it through experiment. The estimated value agreed fairly well with the experimental value. We believe that more experiments will ensure our estimation.

Reference:

[1] T. Kume, K. Enami, Y. Higashi and K. Ueno, J. Jpn. Soc. Prec. Eng. 75, 657 (2009) (in Japanese).