Numerical Simulation of the Contact Area of Smooth Micro-scaled Bodies

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Abstract

The real contact area of non-conforming miniaturised bodies is important for the tribological stress in miniaturized technical systems [1] and for applications in metrology, e.g. tactile micro-probes. Typical wavelengths for form, waviness and roughness leave the conventional technical scale with radii of curvature of 100 µm and less. Proven micro-geometrical models, which use the traditional roughness parameters, are difficult to apply here [2]. Direct numerical simulation of the rough contact is a promising tool to investigate micro-geometrical models. In a first step the numerical model for the elastic bulk deformation is presented. The iterative procedure determines local normal loads of smooth bodies in the elliptic contact. The results are compared with the Hertzian values in a large range of normal loads. The required increments for the numerical model of the rough bodies are derived.

1 Introduction

The Hertz theory [3] is an approximation for the ideal contact situation of two bodies with small displacements, smooth surfaces, low relative curvatures and an isotropic linear elastic material behaviour, where two different materials with Young's modulus $E_{1/2}$ and Poisson's ratio $\nu_{1/2}$ are combined to

$$1/E' = (1-\nu_1)^2/E_1 + (1-\nu_2)^2/E_2 \cdot \tag{1}$$

The normal pressure distribution in the elliptical contact area with the two semiaxes a and b and the maximum Hertzian pressure $p_0=p_H$ in the centre is:

$$p(x,y) = p_0 \cdot \sqrt{1-\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2} \tag{2}$$

see figure 3 (right). Elliptic integrals of the first and second kind have to be solved, see [3,4,5]. With today's powerful computers and numerical routines, no tables or further instruments are required.

2 Data Processing

For the numerical simulation, the elastic half space is used here with the describing surface defined by the distance of the two contacting surfaces sphere and cylinder:

$$z(x,y) = R_S + R_C - \sqrt{R_S^2 - x^2 - y^2} - \sqrt{R_C^2 - x^2}\,, \tag{3}$$

where z is the coordinate normal to the halfspace, R_S the radius of the sphere and R_C the radius of the cylinder. The cylinder main axis is in the direction of the y-coordinate, see figure 3 (right).

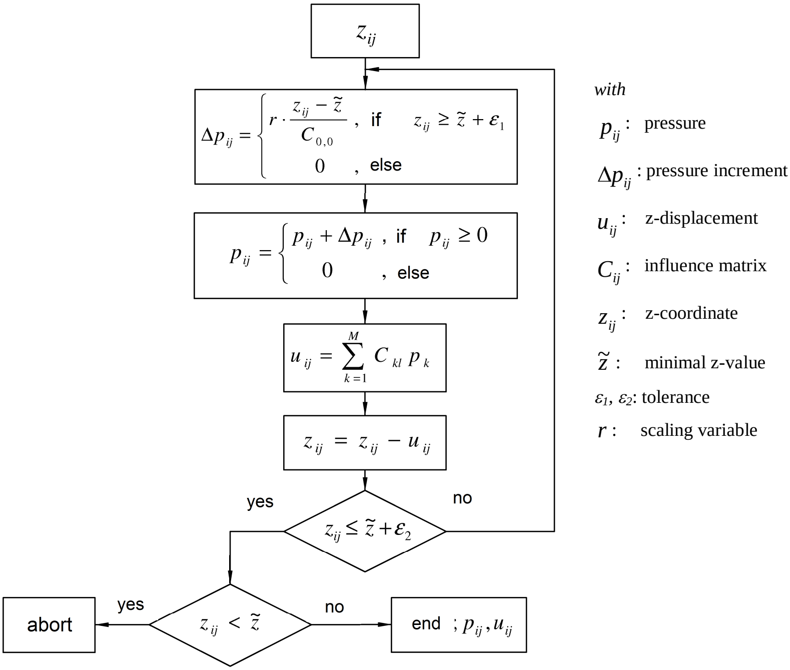

Figure 1: Flow chart of computer program for elastic deformation

Following the Cerutti and Boussinesq solution of the deformation of the elastic halfspace and introducing a discrete surface, the composite displacement in the centre of each of the N surface elements with M elements in initial contact is

$$(\bar{u}_z)_m = \sum_{k=1}^{M} C_{kl}\, p_k \tag{4}$$

with the influence matrix C_{kl} based on Love and the local pressure p_k, see [4,5]. The equation is solved here with a stable converging iteration shown in figure 1. The pressure on each contacting element is determined iteratively and so the deflection of all points on the surface and, hence, the deformed geometry is calculated.
For the computation, the diameter of a sphere is 200 µm, the diameter of the cylinder is 250 µm, Young's modulus 430 GPa and Poisson's ratio 0.25 for both.

3 Results

Figure 2 shows the simulation results with a good agreement of the Hertzian values of the real contact area A_H and of the absolute forces F_H and their numerical approximations A, F respectively. A minimal resolution of the contact radius or the small axis of the contact ellipse of approx. 30 increments is required. The calculated approaches are in the range from 5 nm to 300 nm, the forces to be applied are in the range from 5 mN to 300 mN, and contact radii are from approx. 700 nm to almost 6 µm. The accordance of the results depends on the grade of eccentricity of the contact ellipse for an increasing resolution in the analysed cases.

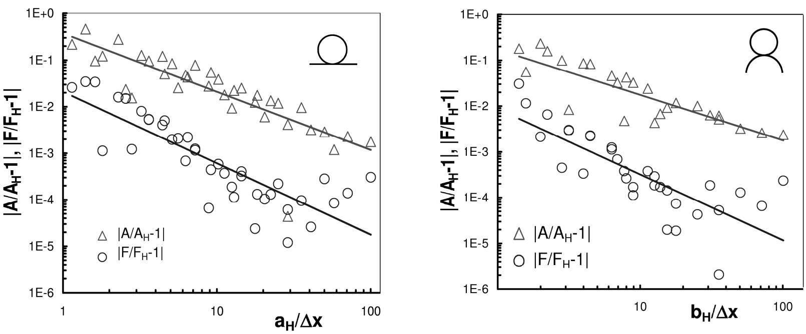

Figure 2: Relative deviations of the numerical solutions as a function of the relative resolution of the contact radii *a* and *b*; Left: Sphere-flat; Right: Sphere-cylinder

As the iteration has pressure as the effect variable, the relative deviation of the simulated pressure is better than 10^{-3} with only 10 elements representing the contact radius, see figure 3.

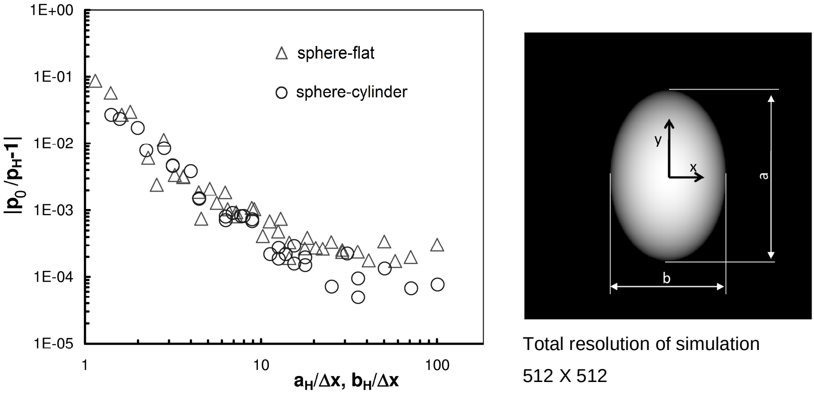

Figure 3: Left: Relative difference of max. local pressure p_0 to the Hertzian value p_H; Right: Grey-scale visualization of pressure distribution and coordinates of eq. 2

4 Discussion and Outlook

A numerical simulation of the elastic contact of smooth bodies is presented, which shows a good agreement with the Hertzian values for a minimal resolution of the contact diameter of approx. 50 elements. In a next step rough surfaces will be implemented to facilitate direct contact simulation under normal load without any micro-geometrical modelling.

References:

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