

# Modeling the Size Effect in Ultra-precision Machining by Mechanism-based Strain Gradient Crystal Plasticity

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## Abstract

In ultra-precision machining, indentation and/or burnishing by the tool-edge of cutting tool occurs. This gives rise to complex elastic and plastic deformation in the machining surface such as materials swelling which is found to be pronounced when the depth cut is small. The size effect observed in ultra-precision machining with a depth of cut from 100 nm and above is simulated by the micro-scratching experiment of FCC single crystals based on the mechanism-based strain gradient crystal plasticity (MSG-CP). The size effect in micro-scratching of an FCC single crystal is studied by measuring the ratio of the thrust force to the projected area of contact between the cutting tool and the work-piece by varying the depth of scratching at the submicron scale. The simulation results demonstrate the effectiveness of the MSG-CP model in studying the scale dependent problems in ultra-precision machining.

## 1 Introduction

To meet the growing demand for the miniaturization of products and three-dimensional features with dimensions ranging from less than a micron to a few tens of microns in a wide range of materials, mechanical micro-machining methods including ultra-precision machining are commonly used. In ultra-precision machining, the depth of cut is within an extremely small fraction of the average grain size of the substrate materials to be cut, and cutting is performed within a single grain. However, the numerical modeling of machining process generally employs phenomenological plasticity without an intrinsic material length scale as the constitutive model, hence ruling out the possibility of considering the size effects in ultra-precision machining. Recently, Liu and Melkote [1] carried out a numerical

simulation of ultra-precision machining processes by employing the strain gradient plasticity put forward by Gao and Huang [2]. Multi-scale modeling by finite element methods and micro-plasticity are needed to characterize the machining processes at different length scales. This is especially true in the fabrication of many of the micro-systems and micro-functional structures as they are of hybrid form, and the modeling of the machining process requires the hierarchical treatment from dislocation dynamics to crystal plasticity.

## **2 Mechanism-based strain gradient crystal plasticity theory (MSG-CP)**

Strain gradient crystal plasticity formulations have been established by many groups [3, 4]. In these continuum models, the different roles that the geometrically necessary dislocations (GNDs) and statistically stored dislocations (SSDs) play are accounted for. The mechanism-based strain gradient crystal plasticity (MSG-CP) proposed by Han et al. [4] is employed in the present simulation. The numerical procedures and implementation details in invoking the UMAT subroutine for conventional crystal plasticity are referred to in Zhang et al. [5]. The incorporation of the strain gradient effect into the conventional crystal plasticity and the materials and constitutive parameters are presented in Lee and Chen [6].

## **3 Results and discussions**

The FE analysis of micro-scratching on a single crystal copper is described below. The dimension of the work-piece is:  $10 \times 10 \times 5 \mu\text{m}^3$  for a sharp conical indenter. The 3D FE mesh is shown in Fig.1a with 11375 nodes and 9408 C3D8 elements. The finer mesh is used in the region underneath the indenter than other part of the specimen to capture an accurate description of deformation and pressure. The half-angle of the conical indenter is  $72^\circ$ . The crystal coordinate system coincides with that of the sample. The bottom surfaces of the work-piece is fixed and all other surfaces are free of constraints. An indentation in the crystallographic direction of  $[0 \ 0 \ 1]$  is performed at point *A* with indentation depth 100 nm and 400 nm, respectively, see Fig.1b, then followed by a scratching to point *B* along the crystallographic direction of  $[1 \ 0 \ 0]$  while keeping the respective depth of indentation unchanged. To characterize the size effect in micro-scratching, the ratio

of the thrusting force to the areas obtained by the projection of total contact areas to a plane normal to the scratching direction is defined here as an appropriate measure, which is expediently termed *thrusting hardness*.

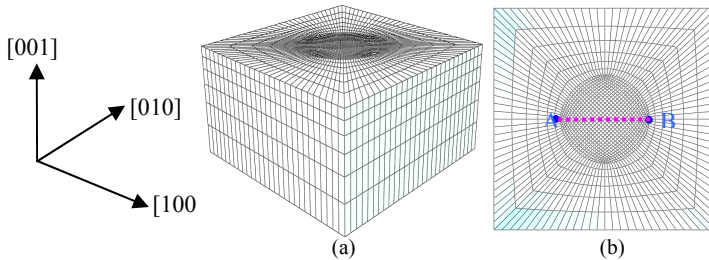


Figure 1 The FE mesh for the scratching simulation for a sharp conical indenter. The number of node and C3D8 element is, respectively, 11375 and 9408 . The scratching is performed in the direction of  $[1\ 0\ 0]$ . The half-angle of the conical indenter is  $72^\circ$  .

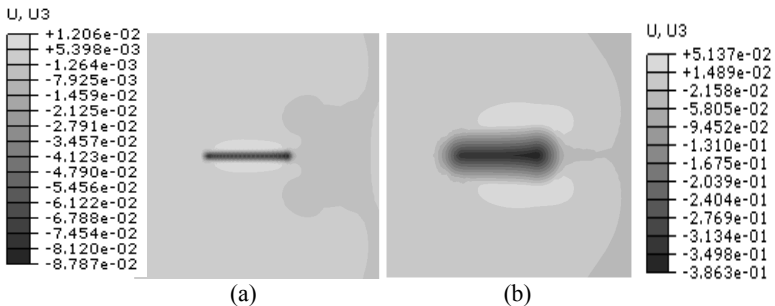


Figure 2 The distribution of displacement components in the direction of  $[0\ 0\ 1]$ . (a) Scratching depth 100 nm, (b) Scratching depth 400 nm.

Figure 2 presents a distribution of displacement components in the direction of  $[0\ 0\ 1]$ , corresponding, respectively, to (a) scratching depth 100 nm, (b) scratching depth 400 nm. Figure 3 shows the calculated thrusting hardness at different scratching depths. It is seen that an increase of thrusting hardness, approximately 500 MPa, is obtained with a decrease of scratching depth from 400 nm to 100 nm. This demonstrates an appreciable size effect in micro-scratching of crystalline materials. At scratching depth 100 nm, a zigzag curve of thrusting hardness versus tool travel step is observed due mainly to the facts that the mesh for the indented material is not dense enough and thus resulting in a relatively large jump in contact area in

consecutive steps. However, such variation can be reduced by the use of finer mesh sizes but this would greatly increase the computing time.

It is noted that the present constitutive model has been calibrated by a direct comparison of the modeling results of Lee and Chen [6] to the experimental findings of McEhlancy [7], who made accurate measurements of hardness versus depth of indentation in copper from 2000 nm down to 100 nm. A good agreement is found. In addition, the phenomena of materials swelling associated with nano-indentation, which have been found to have large effect on the measurement of hardness, are often observed in single point diamond turning and ultra-precision raster milling [8]. Materials swelling generally refers to the elastic-plastic response of work piece when the cutting tool is removed, the effect of which is clearly demonstrated in the simulated results. The MSG-CP model is promising to analyze scale-dependent problems in ultra-precision machining.

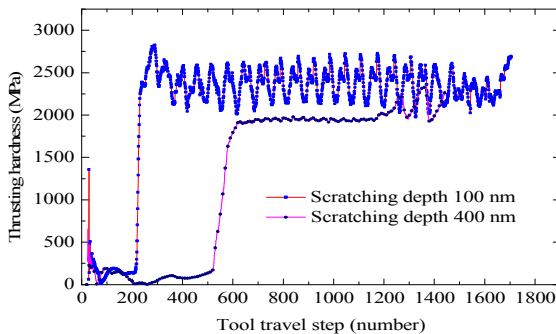


Figure 3 The thrusting hardness corresponding to the scratching depth of 100 nm and 400 nm, respectively.

#### 4 Conclusions

In ultra-precision machining, the surface generation is affected not only by the relative magnitude of the process factors but also by the behavior of the materials at the sub-micrometer level depth of cut that involves complex elastic and plastic deformations such as materials swelling. Materials swelling would cause a tool mark deviated from the theoretical tool profile imprinted on the machined surface, and such effect can neither be explained by the tool feed, machine vibration or the tool nose waviness alone, and is found to be more pronounced with decreasing depth of cut. There is a strong correlation between materials swelling and surface roughness.

This research can yield a better understanding of the size effect, the underlying mechanism of material swelling and its characteristics. Further work will be conducted to predict the cutting force variation and to define the lower limit of surface roughness that can be achieved for a given cutting tool geometry, depth of cut and the known crystallography of the machined surface in micro and nano-machining.

### **Acknowledgement**

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