

# Multivariable Frequency Response Function Estimation of a Micro-Milling Spindle with Active Magnetic Bearings

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## Abstract

In order to use Active Magnetic Bearing (AMB) spindles for monitoring and control of micro-milling, accurate models of the dynamics are needed. Estimation of the Frequency Response Function (FRF) is hampered by the presence of nonlinearities in the AMB. We apply an approach to detecting the presence of nonlinear distortion in the FRF estimate, by means of repeated excitation using orthogonal random phase multisine signals. With this, experiments can be designed that minimize the effect of distortions on the FRF estimates, yielding favorable results.

## 1 Introduction

Active Magnetic Bearing spindle technology is promising for the micro-milling process. Not only are high rotational speeds attainable, also the active nature of these spindles can be used for monitoring and control purposes, resulting in a more stable cutting process and better manufacturing results. Vital in the development of these techniques is the availability of an accurate model of the AMB spindle dynamics. In this contribution data-based modeling of these dynamics is pursued.

It is well known that AMB spindles have various sources of nonlinearities. Examples are the intrinsic nonlinear relationship between coil current, rotor position and electromagnetic force of the magnetic actuators, as well as magnetic hysteresis and eddy current effects. Nonetheless, in many applications a linear model of the AMB spindle dynamics in the operating point is desired. Hence, the problem treated in this paper is to estimate the multivariable Frequency Response Function of the AMB spindle. When such a model is estimated from measured data sequences, careful experiment design is required. Indeed, with small excitation signals the distortion caused by the nonlinearities will generally be small. However, a large variance error in the estimate will occur due to the low Signal-to-Noise Ratio (SNR). On the other

hand, large excitation signals will improve the SNR, but the resulting FRF estimate may be distorted to a large extent due to

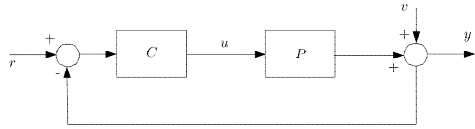


Figure 1 Block diagram of AMB spindle

the nonlinearities. The main question of this paper therefore, is how to detect the level of nonlinear distortion in the FRF estimate of the AMB spindle, in order to be able to make a good trade-off.

## 2 Approach

The AMB spindle setup can be represented by the block diagram in figure 2. In this diagram plant  $P$  represents the  $4 \times 4$  multivariable AMB spindle dynamics, where inputs  $u$  are the control currents and outputs  $y$  the measured positions of the rotor shaft at the AMBs. The controller is denoted as  $C$  and has reference input  $r$ . Here we apply an excitation signal to  $r$  and employ commonly used estimators to find the multivariable FRF of  $P$  from the closed-loop data [1]. For the excitation, orthogonal random phase multi-sine signals are selected. For given excitation power, these excitation signals are known to yield the least variance of the FRF estimate compared to other common excitation signals [2]. In order to answer the main question of this paper, we apply an approach that was introduced in [3] for SISO systems. The key idea of this approach is the following. When a system with nonlinearities (which need to satisfy some formal, but rather unrestrictive conditions) is excited with a random phase multisine signal, then the estimated FRF  $\hat{P}(j\omega)$  can be expressed as

$$\hat{P}(j\omega) = P_0(j\omega) + P_B(j\omega) + P_S(j\omega) + N(j\omega) \quad (1)$$

where  $P_0(j\omega)$  is the linearization of the system in the chosen operating point,  $P_B(j\omega)$  is a nonlinear bias term that varies only with the power spectrum of the excitation signal,  $P_S(j\omega)$  is the so-called stochastic nonlinear noise term that varies with the realization of the random phases of the multisine, and  $N(j\omega)$  is a noise term due to measurement noise. This result can be exploited to detect the presence of nonlinearities by performing experiment repetition. The procedure involves generation of a random phase multisine, and applying multiple periods of the signal to the system. For each period of the multisine an estimate of the FRF can be calculated. This procedure is then repeated for different realizations of the multisine. From eq.

(1) we know that variations of the estimated FRF for different periods of the same realization of the multisine only depends on  $N(j\omega)$ , whereas the variations of the estimated FRF for different realizations depends on both  $P_S(j\omega)$  and  $N(j\omega)$ .

This can be quantified by calculating two variance levels of the estimated FRF, i.e.  $\sigma_{P,j}^2$  and  $\sigma_{P,i}^2$  where the first is the variance of the estimated FRF over periods of the same realization of the excitation, and the second the variance of the estimated FRF calculated over different realizations of the excitation. If both variances are the same,  $P_S(j\omega)$  is zero, and no significant nonlinearities are present. If  $\sigma_{P,i}^2$  is significantly larger than  $\sigma_{P,j}^2$ , this indicates presence of a nonlinear distortion. Being able to detect such distortions, enables us to optimize the excitation spectrum in such a way that the SNR is increased while keeping nonlinear distortions small.

### 3 Results

Applying this approach to the AMB spindle setup in our laboratory, we observe that the nonlinear distortion manifests itself predominantly in the lower frequency regions (see figure 3). With constant excitation spectrum, this results in a standard deviation of the estimated FRF at low frequencies that is up to 50dB larger than at higher frequencies. As a result, the accuracy of the estimated FRF at low frequencies is limited, which is particularly noticeable in the estimates of the cross terms (see figure 4). Improved FRF estimates can be obtained by increasing the excitation amplitude at lower frequencies. Indeed, to generate low frequent position variations of the AMB

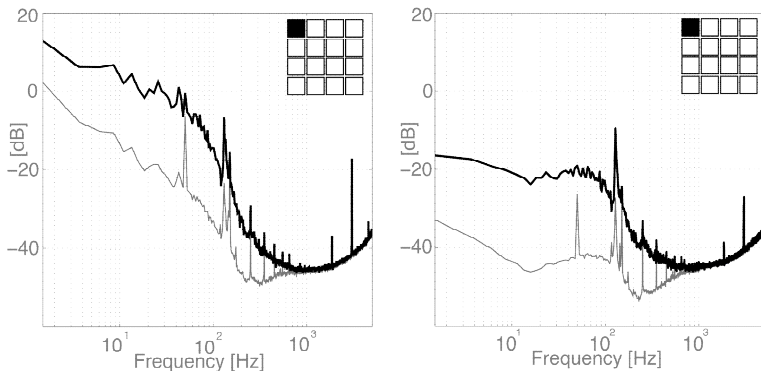


Figure 3 Variance levels of element (1,1) of the FRF (grey  $\sigma_{P,j}$ , black:  $\sigma_{P,i}$ )

Left: excitation with constant amplitude spectrum, Right: excitation with increased amplitude spectrum in lower frequency range.

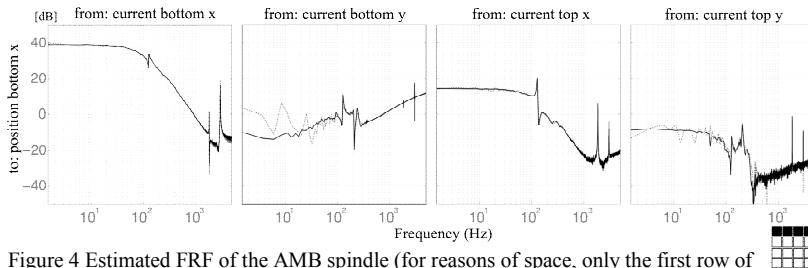


Figure 4 Estimated FRF of the AMB spindle (for reasons of space, only the first row of the FRF matrix is displayed). Grey/dashed: excitation with constant amplitude spectrum, Black/solid: excitation with increased amplitude spectrum in lower frequency range.

spindle, only low force signals are needed. Hence, increasing the excitation amplitude at low frequencies causes the inputs (i.e. the currents) of the AMB spindle only to grow marginally, and consequently the non-linear distortions to remain at approximately the same level. At the same time, larger output signals (i.e. positions) are obtained at these frequencies, so that the effect of the non-linear distortions on the FRF estimates is reduced. Experiments in which the excitation amplitude at lower frequencies is increased up to 25 times confirm this (also see figure 3 and 4). The level of nonlinear distortion is reduced up to 30dB compared to initial experiments with flat excitation amplitude spectrum. The favorable implication of this large reduction is that all cross terms in the multivariable FRF, including the ones with relatively small magnitude, are estimated accurately.

#### 4 Conclusion

An approach to detection of nonlinear distortions in FRF estimates was successfully applied to an AMB spindle. This revealed nonlinear distortion in the lower frequency range. Significant reduction of the effect of the nonlinear distortion on the FRF estimates was obtained by increasing the excitation amplitude at lower frequencies.

#### References:

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- [3] R. Pintelon et al., Experimental Characterization of Operational Amplifiers: a System Identification Approach - part I. IEEE Trans. Instrum. Meas., vol. 53, nr. 3, 2004, 854-862.