

# **A Study of Surface Reconstruction for the Measurement of Ultra-precision Freeform Surfaces**

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## **Abstract**

This paper presents a study of surface fitting techniques to provide a practical method to balance the fitting accuracy and surface smoothness in surface fitting process. The method makes use of the surface intrinsic feature as fitting criteria to minimize the influence of surface roughness such as tool marks to the smoothness of the reconstructed surfaces. The method has been experimentally verified through a series of preliminary measurement experiments. The results show that the developed method can reconstruct ultra-precision freeform surfaces based on measured discrete points, which is not only accuracy enough to characterize the form accuracy of measured objects but also smooth enough to avoid unwanted over fitting error.

## **1 Introduction**

Ultra-precision freeform surfaces are crucial to the development of complex and micro-optical-electro-mechanical devices used in many photonics and telecommunication products such as F-theta lenses for laser printers [1]. It is a fundamental issue in product manufacturing to characterize fabricated products in terms of the form accuracy so as to examine the conformity with designer's requirement. Recently, more and more research work has been done on the use of the surface intrinsic feature such as Gaussian curvature as criteria to perform the localization/registration of measured surface with nominal surface so as to evaluate the form error of the machined surface [2, 3]. However, the calculation of surface intrinsic features of machined surface is highly sensitive to local surface properties such as surface roughness caused by tool marks made during the machining process. This gives rise to surface reconstruction as a crucial means to represent the machined surface from measured discrete points. However, traditional fitting

technologies are inadequate to effectively reconstruct surfaces from cloud of discrete points with appropriate levels of fitness and smoothness which really reflect and represent the real surface definition being determined. As a result, this paper presents a study of surface reconstruction for the measurement of ultra-precision freeform surfaces.

## **2 Surface reconstruction**

Fitting accuracy and surface smoothness usually contradict with each other. If the reconstructed surface is too close to the data points, unwanted variations may occur. On the other hand, if the surface is too smooth, the reconstructed surface cannot meet the requirement for the form accuracy. Instead of finding the best fitting error threshold, a new fitting criterion named Confidence Interval of fitting error is used in the present study to formulate the surface fitting as an optimization process so as to balance the fitting accuracy and surface smoothness. This optimization scheme is highly non-linear and is difficult to solve especially when the required tolerance for approximation is tight. To simplify the process, an initial surface is usually constructed to approximate the points cloud with certain precision [4]. Initial surface is used to estimate the minimal degree of freedom needed to characterize the real shape and to obtain a good parameter value to each data points. Bidirectional sampling strategy developed by author is used to extract two sets of iso-parametric rows of data from points cloud and approximates each row of data to construct a curve network. This curve network is fitted to obtain the initial surface. Then, the squared distance minimization technology is used to optimize the initial surface [5]. To further reduce the uncertainty of the algorithm, a robust statistic method is used to minimize the influence of outliers in the measured data. The process is iterative and is terminated upon achieving desired accuracy.

## **3 Experimental work**

In the present study, the proposed method has been trial implemented by Matlab to validate the proposed fitting criteria, an ideal freeform surface is designed to generate a nominal surface that is obtained with the aid of the surface roughness pattern. The ideal designed surface and the surface roughness pattern are given by Eq. (1) and Eq. (2)

$$z = \sin(0.2x) + \cos(0.3y) \quad (1)$$

$$z = 5 \times 10^{-6} \sin(x) + 5 \times 10^{-6} \cos(x) + 2 \times 10^{-6} \text{rand}(0,1) \quad (2)$$

with the dimension  $0 \leq x \leq 6\pi$  (mm) and  $0 \leq y \leq 6\pi$  (mm). Fig. 1 and Fig. 2 show the extracted cloud of data points and aided error pattern.

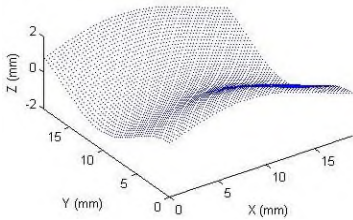


Figure 1: Extracted data points

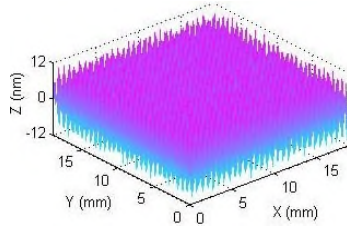
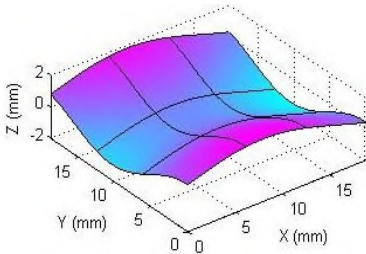
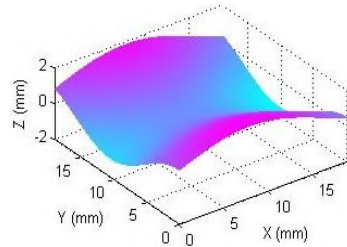


Figure 2: Aided error pattern

The process of the surface reconstruction based on the proposed fitting algorithm is shown in Fig. 3. Fig. 3a) shows the constructed initial surface based on the curve network sampled from cloud of data points, and Fig. 3b) shows the accurate surface which is obtained by optimizing initial surface. Fig. 4 shows the error map.

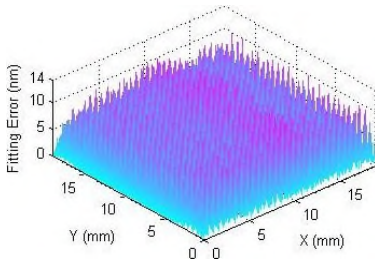


a) Curve network and constructed initial surface

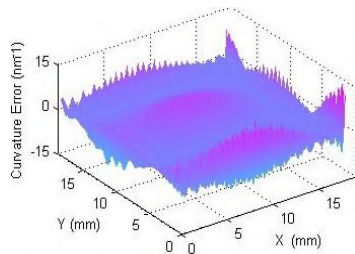


b) Accurate surface

Figure 3: Surface reconstruction process



a) Fitting error



b) Gaussian curvature error

Figure 4: Error map of the constructed surface

The maximum fitting error is 13.39 nm while maximum Gaussian curvature error is  $15.8 \text{ nm}^{-2}$ . It is interesting to note that both fitting accuracy and Gaussian curvature are in high level, which infers that the fitting accuracy and the smoothness are well balanced for this surface fitting.

#### **4 Conclusion**

In this paper, a novel criteria based surface fitting and reconstruction method is studied for the reconstruction of ultra-precision freeform surfaces from clouds of scanned 3D data, which is deduced by considering the influence of the roughness of fabricated surfaces to the intrinsic feature of reconstructed surfaces such as Gaussian curvature. Preliminary experimental work has been conducted to verify the performance of the developed method. The results show that the proposed method is able to provide an effectiveness means for increasing the accuracy of the fitting of measured surfaces in the characterization of ultra-precision freeform surfaces. The possible outcome of the proposed study will contribute to the representation and characterization of ultra-precision freeform surfaces.

#### **Acknowledgement**

The work described in this paper was mainly supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. PolyU 5141/08E). The authors would also like to express their sincere thanks to the Research Committee of The Hong Kong Polytechnic University for their financial support for the project.

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