Free Spectral Range Measurement of Fabry-Perot Cavity Using Transmission Light, Single Frequency Modulation and Null Method Under Off-resonance Condition

M. Aketagawa, T. Yashiki, S. Kimura, H. Iwata and T. Banh Quoc

Nagaoka University of Technology, Japan

masatoaa@vos.nagaokaut.ac.jp

Abstract

In this paper, we propose a novel method to measure the free spectral range (FSR) of a Fabry-Perot cavity (FPC) using frequency modulation (FM) with an electric optical modulator (EOM) and the null method. A laser beam modulated by the EOM, to which a sine-wave signal is supplied from a radio frequency (RF) oscillator, is incident on the FPC. The transmission light from the FPC is observed and converted to an RF signal by a high-speed photodetector, and the RF signal is synchronously demodulated with a lock-in amplifier (LIA) by referring to the oscillator. We theoretically and experimentally demonstrate that the LIA signal become null with a steep slope, when the modulation frequency equals the FSR under the condition that the carrier frequency of the laser is slightly detuned from the resonance of the FPC.

1 Intorduction

In the future, an absolute length measurement method with a measurement range of 1 meter or more and an accuracy of sub-nanometer order or less will be required, due to the progress of nanotechnology and ultraprecision engineering. To overcome this problem, many researchers have developed absolute length measurement methods with a small measurement uncertainty[1],[2],[3]. Since the free spectral range (FSR) of a Fabry-Perot cavity (FPC) is inversely proportional to the optical distance between the two mirrors of the FPC, the FSR measurement method may be an alternative candidate for absolute length measurements[2],[3]. In this paper, a novel FSR measurement method using the interference between a carrier and frequency modulation (FM) sidebands is proposed. In our method, the FSR can be determined from the null method using transmission light and the FM technique under the condition that the carrier frequency is slightly detuned from the resonance of the FPC.

2 Measurement principle

A conventional FPC consists of two mirrors separated by a distance of L. Its FSR ($f_{FSR} = \omega_{FSR}/2\pi$) is defined as

$$f_{FSR} = \frac{\omega_{FSR}}{2\pi} = \frac{c}{2n_c L} \quad (1),$$

where c and n_c are the velocity of light in a vacuum and the refractive index of the medium inside the cavity. If the f_{FSR} or ω_{FSR} is measured, then the absolute optical length ($n_c L$) can be determined using Eq. (1). Fig. 1 shows the basic FSR-measuring system discussed in this paper. In the figure, ECLD, EOM, PD, OSC and LPF are an extra cavity laser diode, an electric optical modulator, a photodetector, an oscillator and a low-pass filter, respectively. A lock-in amplifier (LIA) consists of a mixer and the LPF.

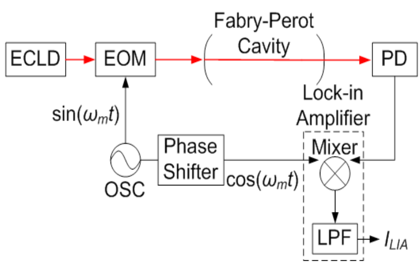

Fig. 1 Basic FSR measuring system.

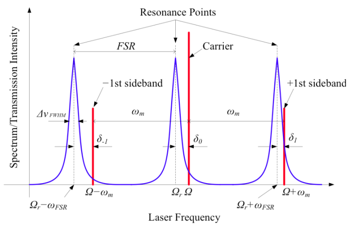

Fig. 2 Relationship between FPC transmission, carrier (0-th) and FM sidebands (±1th).

The electric field $E_{EOM}(t)$ modulated by the EOM is represented as

$$E_{EOM}(t) = E\left[\sum_{n=-\infty}^{\infty} J_n(m)\exp[i(\Omega + n\omega_m)t\right] \quad (2),$$

where E, Ω, m and J_n are the original electric field of the ECLD, its carrier frequency, the modulation depth and the n-th order Bessel function, respectively. Fig. 2 shows the relationship between FPC transmission, carrier and two FM sidebands. In Fig. 2, Ω_r and $\Delta\nu_{FWHM}$ are the main resonance frequency closest to the carrier frequency Ω and the full width at half maximum of the resonance curve, respectively. As shown in Fig. 2, the detuning frequency shifts δ_n (n =0, ±1, ±2, ...) between resonance frequencies and the FM sidebands can be written:

$$\delta_n = (\Omega + n\omega_m) - (\Omega_r + n\omega_{FSR}) \\ = \delta_0 + n(\omega_m - \omega_{FSR})$$

(3),

where δ_0 is $(\Omega - \Omega_r)$. Eq. (3) can be rewritten by normalizing it relative to the ω_{FSR} as follows:

$$\bar{\delta}_n = \bar{\delta}_0 + n(\bar{\omega}_m - 1)$$

(4),

where $\bar{\delta}_n = \delta_n / \omega_{FSR}$, $\bar{\delta}_0 = \delta_0 / \omega_{FSR}$, $\bar{\omega}_\mathrm{m} = \omega_m / \omega_{FSR}$.

When an arbitrary optical field E_i is incident on a lossless FPC, the transmission field E_t is approximately given by [3],[4]

$$E_t = \frac{E_i}{1 + 2iF\bar{\delta}} = T_{FPC}(\bar{\delta})E_i \qquad (T_{FPC}(\bar{\delta}) = \frac{1}{1 + 2iF\bar{\delta}}) \tag{5}$$

where $\bar{\delta}$, F and T_{FPC} are the normalized detuning frequency from the cavity resonance, the cavity finesse and the amplitude transmission function of the FPC, respectively. If the electric field $E_{EOM}(t)$ given by Eq. (2) is incident on the FPC, then, using Eq. (5), the transmitted electric field $E_{FPC}(t)$ is given by

$$E_{FPC}(t) = E \sum_{n=-\infty}^{\infty} [J_n(m) T_{FPC}(\bar{\delta}_n) \exp\{i(\Omega + n\omega_m)t\}] \tag{6}$$

where a dispersion effect of the FPC is neglected. Therefore, the transmitted intensity $I_{FPC}(t)$ is written as

$$\begin{aligned} I_{FPC}(t) &= |E|^2 \sum_{n=-\infty}^{\infty} J_n(m)^2 |T_{FPC}(\bar{\delta}_n)|^2 \\ &+ 2|E|^2 \sum_{n=0}^{\infty} [J_n(m) J_{n+1}(m) \{ \mathrm{Re}[T_{FPC}(\bar{\delta}_n) T^*{}_{FPC}(\bar{\delta}_{n+1}) - T^*{}_{FPC}(\bar{\delta}_{-n}) T_{FPC}(\bar{\delta}_{-n-1})] \cos\omega_m t \\ &\qquad + \mathrm{Im}[T_{FPC}(\bar{\delta}_n) T^*{}_{FPC}(\bar{\delta}_{n+1}) - T^*{}_{FPC}(\bar{\delta}_{-n}) T_{FPC}(\bar{\delta}_{-n-1})] \sin\omega_m t \}] \\ &+ (\text{higher than } 2\omega_m \text{ terms}) \end{aligned} \tag{7}$$

The LIA signal I_{LIAc} for $\cos\omega_m t$ term is represented as,

$$I_{LIAc} = k_{LIA} |E|^2 \sum_{n=0}^{\infty} [J_n(m) J_{n+1}(m) \{ \frac{1 + 4\boldsymbol{F}^2 \bar{\delta}_n \bar{\delta}_{n+1}}{(1 + 4\boldsymbol{F}^2 \bar{\delta}_n{}^2)(1 + 4\boldsymbol{F}^2 \bar{\delta}_{n+1}{}^2)} - \frac{1 + 4\boldsymbol{F}^2 \bar{\delta}_{-n} \bar{\delta}_{-n-1}}{(1 + 4\boldsymbol{F}^2 \bar{\delta}_{-n}{}^2)(1 + 4\boldsymbol{F}^2 \bar{\delta}_{-n-1}{}^2)} \}] \tag{8}$$

where k_{LIA} is the amplification constant of the LIA. The LIA signal I_{LIAc} must cross the null when the modulation frequency equals the FSR of the FPC and the carrier frequency is slightly detuned from the resonance. The optimum carrier detuning shift (δ_0) for the steepest slope is approximately 29% of $\Delta\nu_{FWHM}$.

3 Experimental results

In the experiment, we used the ECLD (NewFocus 6304, center wavelength = 633 nm, power =1 mW), the EOM (JEOPTIK Laser PM633), the OSC (Anirtsu 3641A), the FPC (Neoark 2221), an avalanche PD (Hamamatsu Photonics C5658), LIA (=a passive double-balanced mixer (R&K MX130-0S) + an LPF (NF Corporation E-3201, cut-off frequency = 1 kHz)). The modulation frequency of the OSC was measured by a frequency counter (Pendulum CNT-90, not shown in Fig. 1). The LIA signal and the modulation frequency were simultaneously recorded with a personal computer. The FSR and finesse of the FPC in a normal temperature of 293 K in the specification are approximately 888.9 MHz and 142, respectively. In the experiment, the modulation depth *m* was adjusted to approximately 1.08 rad.

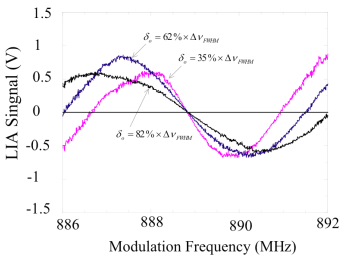

Fig. 3 Relationships between modulation frequency and LIA signal. Frequency scan step = 20 kHz/step.

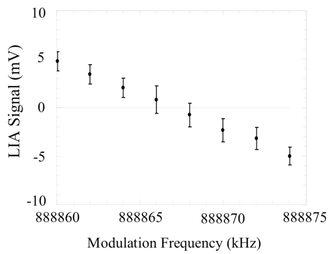

Fig. 4 Relationship between modulation frequency and LIA signal. Frequency scan step = 2 kHz/step.

Fig. 3 shows the measured relationships between the modulation frequency and the LIA signal I_{LIAc}. In Fig. 3, the results for the three detuning frequency shifts (δ_0), 35, 62 and 82% of $\Delta\nu_{FWHM}$, are shown. Fig. 4 shows the detailed relationship between the modulation frequency and the LIA signal I_{LIAc} for the case of a detuning shift (δ_0) of 35% of $\Delta\nu_{FWHM}$. From Fig. 4, the FSR was determined to be 888867 kHz (n_cL =168.637 mm). The estimated uncertainty was 10^{-6} order.

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