Joint Special Interest Group meeting between euspen and ASPE
Advancing Precision in Additive Manufacturing
KU Leuven, Belgium, September 2023
www.euspen.eu

Triangle mesh based 3D surface characterisation for additive manufacturing parts inspection

W. Zeng[1,*], S. Lou[1], F. Blateyron[2], P. J. Scott[1], X. Jiang[1]

[1]*EPSRC Future Metrology Hub, Centre for Precision Technologies, University of Huddersfield, Huddersfield, HD1 3DH, UK*
[2]*Digital Surf, 16 rue Lavoisier, 25000 Besançon, France*

[*]*z.wenhan@hud.ac.uk*

Abstract
Additive manufacturing (AM) produces components with intricate geometries, such as complex shapes, lattice structures, and 3D surface topography featuring re-entrant features and undercuts. X-ray computed tomography (XCT) is a valuable tool for measuring unique characteristics of AM surfaces, including particle features, surface cracks, and open porosity. Understanding the impact of these features is essential for predicting the performance of AM parts in fatigue, heat exchange, and osseointegration applications. However, characterising these XCT-measured features presents significant challenges. Firstly, their complex base or reference surfaces cannot be adequately described using Euclidean-space-based methods. Secondly, the surfaces measured are typically represented by polygon meshes rather than the lattice grid data used by conventional surface measurement instruments. As a result, standard surface characterisation techniques based on regular lattice grids, such as profile/areal surface parameters, form removal through fitting, filtration (Gaussian, spline, and robust filtration), and feature characterisation, are not directly applicable to these surfaces. This paper introduces the triangle mesh-based 3D surface characterisation method developed by CPT in collaboration with Digital Surf. The method encompasses a general characterisation framework, 3D surface parameters, freeform mesh fitting, and the latest advancements in triangle mesh-based filtration for separating the reference mesh and SL surface to analyse surface parameters.

Keywords: 3D surface characterisation, triangle mesh surface, AM surface inspection

1. Introduction

Complex freeform surfaces find extensive applications across various industries, including aerospace, automobile, bio-engineering, medical, and consumer electronics. However, characterising these surfaces presents a notable challenge due to their intricate base surface (reference surface/mean surface), which cannot be adequately described using Euclidean-space-based methods like the lattice grid. An exemplary technology that showcases the complexity of these surfaces is additive manufacturing (AM), which allows the creation of highly intricate geometries through layer-by-layer material addition, surpassing the limitations of traditional manufacturing processes. To measure these complex surfaces, non-conventional instruments such as 3D cameras, laser trackers, and X-ray computed tomography are utilised [1,2,3]. The resulting measurement data is often represented as a polygon mesh, and the underlying domain typically possesses non-Euclidean characteristics. Consequently, conventional surface characterisation techniques like Fourier analysis, Gaussian filters, spline filters, and lattice-grid-based surface parameters are incompatible with such surfaces [4]. In this paper, we introduce a novel methodology proposed by CPT for characterising complex freeform surfaces based on triangle meshes [4,5]. The methodology comprises a comprehensive characterisation framework that encompasses 3D surface parameters, freeform mesh fitting, and the latest advancements in triangle mesh-based filtration. These advancements enable the separation of the reference mesh and SL surface, facilitating in-depth analysis of surface parameters.

2. General framework for Mesh Surface Characterisation

Figure 1 presents an overarching framework for characterising complex freeform surfaces based on triangle meshes. The characterisation and parameterisation of surface texture are accomplished through three fundamental steps: surface sampling and representation, decomposition (including association and filtration), and parameterisation. However, the characterisation and parameterisation of surface texture for freeform geometries represented by triangle meshes pose notable challenges. These challenges necessitate a thorough reconsideration and reimagining of each individual step within the characterisation process [5,10].

The initial step, surface sampling and representation, involves carefully selecting sampling points on the surface and accurately representing the surface using a triangle mesh. For XCT measurement, the challenges is how to extract the surface mesh from the volumetric image.

The subsequent step, decomposition, comprises various sub-steps, such as mesh-based association and mesh-based filtration. It is one of the key steps, similar to conventional surface characterisation, for separating the surface texture, features, and patterns from the underlying form/geometry.

Finally, the parameterisation step is to define quantitative measures and descriptors that effectively capture the essential aspects of the surface texture. These parameters provide valuable insights into the surface properties and facilitate further analysis and comparison of different surfaces. Under this framework, we have developed mesh-based freeform surface parameters.

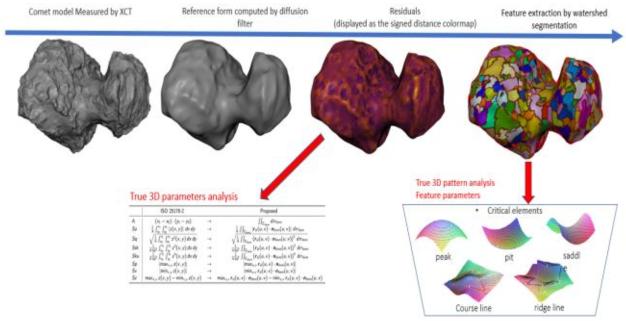


Figure 1. The general framework for the Triangle-mesh based Complex Surface characterisation [5]

3. 3D Surface mesh association

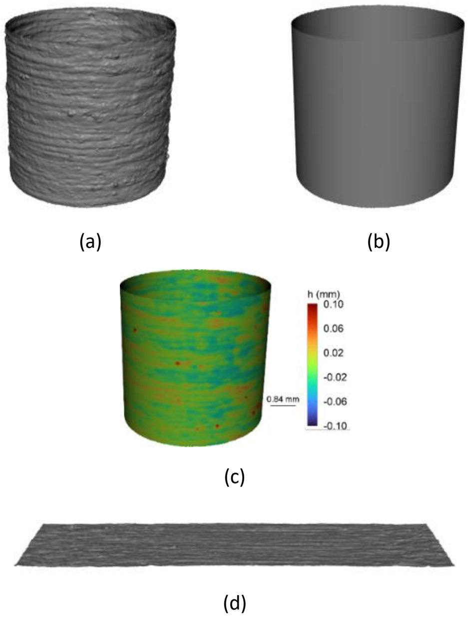
For a triangle mesh surface, when the nominal form is known and fixed, the reference surface mesh can be calculated using the least-squares optimisation to best fit the nominal form. The surface texture is obtained by calculating the perpendicular vector from the reference surface to the measured mesh surface. The mesh surface is therefore decomposed into two parts:

$$\boldsymbol{r}(u,v) = \boldsymbol{r}_{form}(u,v) + r_{sl}(u,v) \cdot \boldsymbol{n}_f(u,v) \tag{1}$$

where $\boldsymbol{r}_{form}(u,v)$ represents the reference form surface, and $\boldsymbol{n}_f(u,v)$ is the normal of the form mesh and $r_{sl}(u,v)$ is a scalar field representing the scale-limited surface. The form is estimated by computing the parameters of the primitive according to the weighted total least squares method:

$$\widehat{\boldsymbol{\beta}} = arg\ min_{\boldsymbol{\beta}} \sum_{i=1}^{n} w_i \cdot (\boldsymbol{x}_i - \boldsymbol{f}(\boldsymbol{\beta}, \boldsymbol{x}_i))^2 \tag{2}$$

where $\boldsymbol{\beta}$ is the vector of parameters that define the reference form, $\boldsymbol{x}_i$ is a point of the measured surface, and $\boldsymbol{f}(\boldsymbol{\beta}, \boldsymbol{x}_i)$ is the value of the point $\boldsymbol{x}_i$ projected on the function, and here the weight assigned to each vertex on the mesh is computed as the total area of the one-ring neighbourhood triangles. The best-fit algorithm undergoes two stages: an initial coarse approximation fit, followed by a more optimised fit [5].



(a) (b)

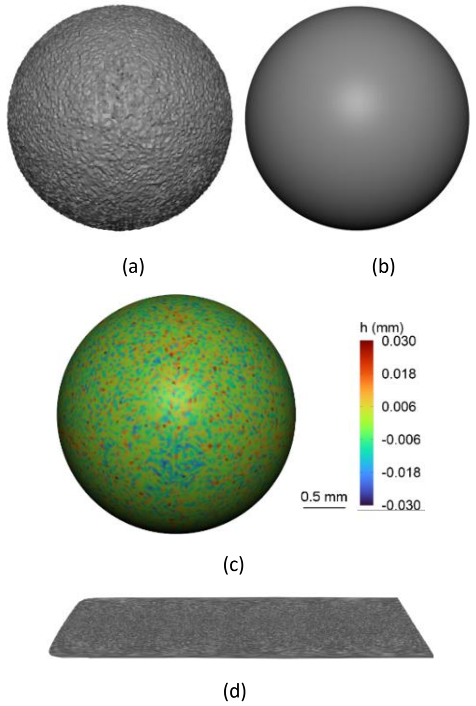
(c)

(d)

Figure 2. 3D mesh fitting of an AM cylindrical surface: (a) measured mesh; (b) fitted reference mesh(form); (c) surface texture (displayed as the signed distance colormap; (d) unwrapped surface texture

Figure 2 gives an example of using the weighted least square to fit a cylindrical shape AM part surface represented by triangle-mesh and measured by XCT. A cylinder can by defined by a point on the axis $\boldsymbol{x}_0$, the axis direction $\boldsymbol{a}$ and the radius r, the equation to minimise becomes:

$$\min_{r,\boldsymbol{x}_0,\boldsymbol{a}} \sum_{i=1}^{n} w_i \cdot [r - \|\boldsymbol{x}_i - \boldsymbol{x}_0 + (\boldsymbol{x}_i - \boldsymbol{x}_0) \cdot \boldsymbol{a}\|]^2 \tag{3}$$



(a) (b)

(c)

(d)

Figure 3. 3D fitting of an AM sphere surface: (a) measured mesh; (b) fitted reference mesh(form); (c) surface texture (displayed as the signed distance colourmap; (d) unwrapped surface texture

Figure 3 gives another example of using the weighted least square to fit a spherical shape AM part surface represented by triangle-mesh. A sphere can be defined by the centre $\boldsymbol{x}_0$ and the radius r, the objective function becomes:

$$\min_{r,\boldsymbol{x}_0} \sum_{i=1}^{n} w_i \cdot (r - \|\boldsymbol{x} - \boldsymbol{x}_0\|)^2 \tag{4}$$

Compared with fitting with the cylinder case, the unwrapped the surface texture has distortion. However, this unwrap operation is only for demonstration, for the parameter calculation, all the parameters calculated actually is based on the smoothed manifold and the residual.

4. 3D Surface mesh filtration

In the case that the nominal form is not known or cannot be represented by a function, the filtration operation can be used to determine the reference surface. CPT has proposed using Laplacian diffusion filter. The Laplacian diffusion filter can be defined as:

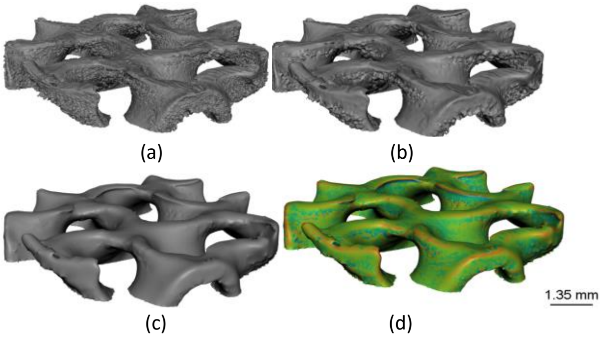
$$\frac{\partial f(p,t)}{\partial t} - \Delta f(p,t) = 0 \tag{5}$$

where p is the position of the point, Δ denotes the laplace-beltrami operator, which is the generalisation of Laplacian operation to function defined on Riemannian manifolds. The proposed method applies a diffusion process iteratively on the measured triangle mesh to generate a smoothed version that serves as the reference form for evaluating geometry and dimensions. The measured vertices are then orthogonally projected onto the smoothed mesh, and the distance between them is regarded as the surface texture. For most practical applications, the geometry is represented by a three-dimensional triangular mesh formed from discrete data points. In such cases, there are a number of numerical methods available for approximating both the Laplace–Beltrami operator and solutions of the diffusion equation at discrete time values.

$$\Delta_M f(\boldsymbol{x}_i) = \frac{1}{2\,A_M(\boldsymbol{x}_i)}\sum_{j\in N(i)}\left(\cot\alpha_{ij} + \cot\beta_{ij}\right)\left(f(\boldsymbol{x}_j) - f(\boldsymbol{x}_i)\right) \quad (6)$$

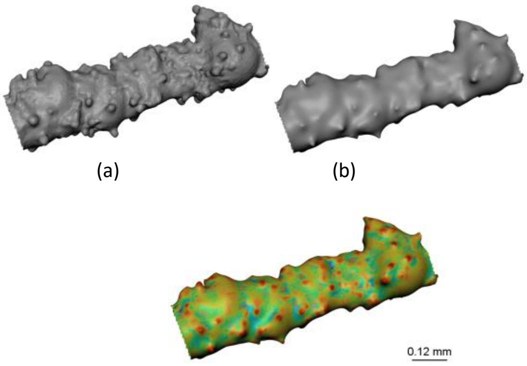
$$\Delta_M f(\boldsymbol{x}_i) = \frac{3}{2\,A(\boldsymbol{x}_i)}\sum_{j\in N(i)}\left(\cot\alpha_{ij} + \cot\beta_{ij}\right)\left(f(\boldsymbol{x}_j) - f(\boldsymbol{x}_i)\right) \quad (7)$$

Where, $A(\boldsymbol{x}_i)$ is the area of all triangles that contain vertex $\boldsymbol{x}_i$, and the angles α_{ij} and β_{ij} are the two angles opposing the edge $[\boldsymbol{x}_i, \boldsymbol{x}_j]$ of the two triangles sharing the edge. $A_m(\boldsymbol{x}_i)$ is the area of the region formed by joining the circumcentres of all triangles containing vertex $\boldsymbol{x}_i$.



(a) (b) (c) (d)

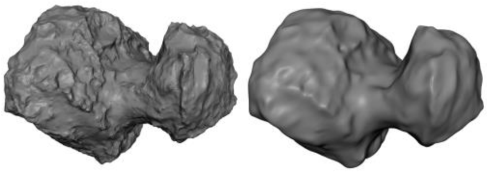
Figure 4. Laplacian diffusion filter on AM lattice structure: (a) measured mesh; (b) filtered reference mesh with cutoff (iteration=100); (c) reference mesh with cutoff (iteration=500); (d) surface texture (displayed as the signed distance colormap)



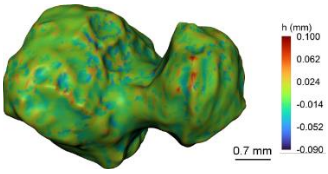
(a) (b) (c)

Figure 5. Laplacian diffusion filter on AM parts: (a) measured mesh; (b) filtered reference mesh; (c) surface texture (displayed as the signed distance colourmap)

Figure 4 and figure 5 show examples that using the proposed Laplacian diffusion filter to separate lattice structure and their surface texture.



(a) (b)



(c)

Figure 6. Laplacian diffusion filter on AM reproduced comet: (a) measured mesh; (b) filtered reference mesh; (c) surface texture (displayed as the signed distance colourmap)

Figure 6 shows the example of triangular mesh data of 67P/Churyumov–Gerasimenko Comet obtained from the Rosetta mission. The proposed filter is used for the pre-step for watershed segmentation to identify feature regions.

5. 3D Surface Parameters

The 3D surface texture parameters recently developed by CPT allow X-ray computed tomography (XCT) measurement data to be used to assess 3D surface topography. Currently, the developed 3D parameters include height parameters, hybrid parameters, volume parameters and part of feature parameters based on 3D watershed segmentation.

5.1. 3D Height Parameters

Height parameters are used to represent an overall measure of surface texture. Unlike the conventional areal counterparts, the 3D height parameters account for re-entrant features in their computation [6,7]. The 3D extension of four commonly used 3D height parameters are

$$Sa = \frac{1}{A_{form}}\iint_{\Sigma_{form}} |r_{sl}(u,v)|\; d\sigma_{form} \quad (8)$$

$$Sq = \sqrt{\frac{1}{A_{form}}\iint_{\Sigma_{form}} r_{sl}^2(u,v)\; d\sigma_{form}} \quad (9)$$

$$Ssk = \frac{1}{A_{form}\cdot Sq^3}\iint_{\Sigma_{form}} r_{sl}^3(u,v)\; d\sigma_{form} \quad (10)$$

$$Sku = \frac{1}{A_{form}\cdot Sq^4}\iint_{\Sigma_{form}} r_{sl}^4(u,v)\; d\sigma_{form} \quad (11)$$

Where $d\sigma_{form} = \left\|\boldsymbol{r}_{form,u}(u,v) \times \boldsymbol{r}_{form,v}(u,v)\right\| du\, dv$, and $\boldsymbol{r}_{form,i}(u,v)$ is the partial derivative of $\boldsymbol{r}_{form}(u,v)$ in the i direction, $d\sigma_{form}$ is the infinitesimal areal element and $A_{form} = \iint_{\Sigma_{form}} d\sigma_{form}$ is the area of the form surface.

5.2. 3D Hybrid Parameters

Hybrid parameters incorporate both height and spatial information. Two hybrid parameters Sdr (Developed interfacial area ratio) and Srf (percentage of the re-entrant features) are proposed[6,7].

$$Sdr = \frac{A - A_{form}}{A_{form}} \quad (12)$$

$$Srf = \frac{A_{form} - A_{shadow}}{2\cdot A_{shadow}} \quad (13)$$

where A is the area of the actual surface, A_{form} is the area of the form surface, and A_{shadow} is the area of the shadow on the form surface.

5.3. 3D Material Ratio Curves and Volume Parameters

A set of 3D Material Ratio Curves and Volume Parameters are developed [7]. An exciting feature of the 3D material ratio curve is its ability to signal the re-entrant features represented by the curve's recesses. Another helpful function is to provide rich information on the volume density of surface peaks (e.g. protruding particles) and surface valleys (e.g. surface pores)[8].

5.4. 3D Watershed Segmentation and Feature Parameters

Feature parameters consider the topographical features on the surface, which are identified by the watershed segmentation. It is reported that they are more relevant to functional performance [9]. To enable the computation of 3D feature parameters, traditional watershed segmentation based on height maps is extended to the 3D mesh case [9]. Figure 7 illustrates an example of the 3D pit-based watershed segmentation of a lattice strut.

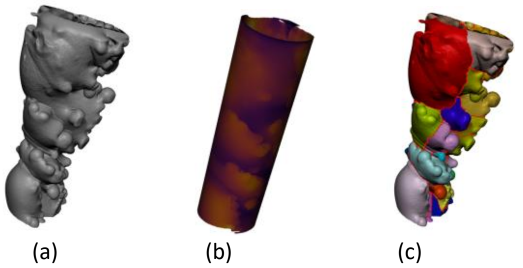
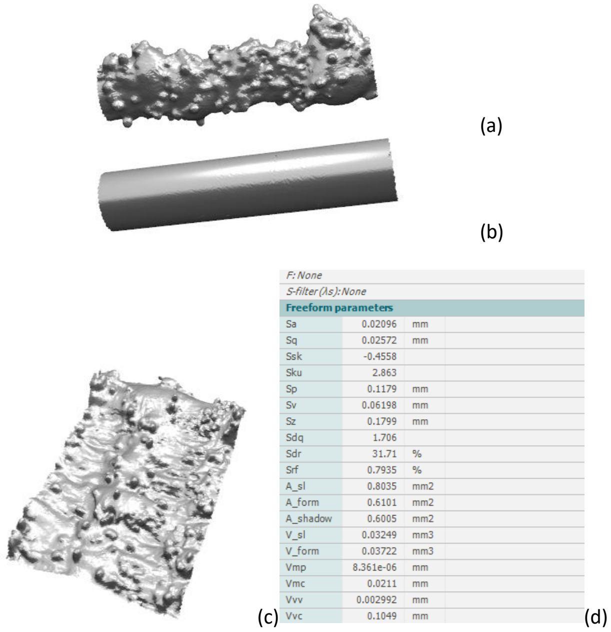


Figure 7. Feature segmentation of an AM lattice strut: (a) measured mesh; (b) surface texture (displayed as the signed distance colourmap); (c) feature segmentation

5.5 software package

Some of the above discussed techniques, such as the mesh association, field parameters, material ratio related parameters, have been coded as software add-ons and included in Digital Surf's Mountains 9, as shown in figure 8.



F: None

S-filter (λs): None

Freeform parameters			
Sa	0.02096	mm	
Sq	0.02572	mm	
Ssk	-0.4558		
Sku	2.863		
Sp	0.1179	mm	
Sv	0.06198	mm	
Sz	0.1799	mm	
Sdq	1.706		
Sdr	31.71	%	
Srf	0.7935	%	
A_sl	0.8035	mm2	
A_form	0.6101	mm2	
A_shadow	0.6005	mm2	
V_sl	0.03249	mm3	
V_form	0.03722	mm3	
Vmp	8.361e-06	mm	
Vmc	0.0211	mm	
Vvv	0.002992	mm	
Vvc	0.1049	mm	

Figure 8. Snapshot of the software add-on: (a) measured mesh; (b) filtered reference mesh; (c) surface texture (displayed as the signed distance colourmap); (d) 3D parameters table

6. Conclusions

Addtively Manufactured parts with complex shapes have been widely used in aerospace, automobile, bio-engineering, medical and consumer electronics etc., however, Compared with traditional 'stochastic surfaces', one of the significant challenges for the characterisation is that they have complex base surface (reference surface/mean surface), which cannot be described using Euclidean-space (such as the lattice grid) based method. These complex surfaces are measured by non-conventional instruments, and the measurement data is represented by a polygon mesh, which cannot be processed by traditional surface characterisation techniques, such as Fourier analysis, Gaussian filters, spline filters, lattice-grid based surface parameters, etc. In the last ten years, CPT has developed the mathematical fundaments of triangle mesh-based freeform surface characterisation, and in collaboration with Digitial Surf, we are developing a methodology comprising a comprehensive characterisation framework that encompasses 3D surface parameters, freeform mesh fitting, and the latest advancements in triangle mesh-based filtration. These advancements enable the separation of the reference mesh and SL surface, facilitating in-depth analysis of surface parameters. The developed method is not only for triangle mesh surface texture analysis but also has great potential for dimension/geometry analysis as well as internal defects analysis.

Acknowledgements

The authors would like to acknowledge the support of the UK's Engineering and Physical Sciences Research Council (EPSRC) Future Advanced Metrology Hub (EP/P006930/1).

References

[1] Leach R K, Bourell D, Carmignato S, Donmez A, Senin N And Dewulf W 2019 Geometrical metrology for metal additive manufacturing *CIRP annals* **68**(2) 677-700

[2] Du Plessis, A. and Beretta, S., 2020. Killer notches: The effect of as-built surface roughness on fatigue failure in AlSi10Mg produced by laser powder bed fusion. Additive Manufacturing, 35, p.101424.

[3] Sun W, Giusca C, Lou S, Yang X, Chen X, Fry T, Jiang, X, Wilson A, Brown S and Boulter H 2022 Establishment of X-ray computed tomography traceability for additively manufactured surface texture evaluation Additive Manufacturing **50** 102558

[4] Jiang X, Cooper P and Scott P J 2011 Freeform surface filtering using the diffusion equation, *Proceedings: Mathematical, Physical and Engineering Sciences* **467**(2127) 41-859

[5] Jiang X and Scott P J 2020 Advanced Metrology: Freeform Surface, Academic Press, ISBN 978012817337

[6] Pagani L, Qi Q, Jiang X and Scott,P J 2017 Towards a new definition of areal surface texture parameters on freeform surface. *Measurement* **109** 281-291

[7] Pagani L, Townsend A, Zeng, W, Lou S, Blunt L, Jiang X Q and Scott P J 2019 Towards a new definition of areal surface texture parameters on freeform surface: Re-entrant features and functional parameters *Measurement* **141** 442-459

[8] Lou S, Zhu Z, Zeng W, Majewski C, Scott P J and Jiang X 2021 Material ratio curve of 3D surface topography of additively manufactured parts: an attempt to characterise open surface pores *Surface Topography: Metrology and Properties* **9**(1) 015029.

[9] Lou S, Pagani L, Zeng W, Jiang X and Scott PJ 2020 Watershed segmentation of topographical features on freeform surfaces and its application to additively manufactured surfaces *Precision Engineering* **63** 177-186

[10] Zeng W, Lou S, Blateyron F, Scott P and Jiang X 2022 A General Framework for the Characterisation of Triangle Mesh based Freeform Surface and its Applications in Additive Manufacturing Metrology, *Met & Props 2022*, Glasow